

## Stat 218 - Day 15

### Comparing Two Means: Confidence Interval

Today and all this week we will consider statistical issues of *comparing two groups*. This is an especially important issue because analyze the results of experiments often involves comparing two groups. We will again turn our attention back to *quantitative* variables this week.

Recall from last week that the general form of a confidence interval is:

(point estimate)  $\pm$  (critical value)  $\times$  (standard error of point estimate).

Now we will estimate the *difference between two population means*.

Notation:

$\mu_i$  = population mean for group  $i$

$n_i$  = sample size for group  $i$

$\bar{y}_i$  = sample mean for group  $i$

$s_i$  = sample standard deviation for group  $i$

We will form a confidence interval for the difference in population means  $\mu_A - \mu_B$  by:

$(\bar{y}_A - \bar{y}_B) \pm t^* \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}$ . There are several suggested ways to calculate the degrees of freedom:

we will use a conservative rule that says to use the smaller of  $n_A - 1$  and  $n_B - 1$ . The technical conditions required for the validity of this procedure are that:

- 1) the data can be regarded as independent random samples
- 2) the underlying populations follow normal distributions, *or* the sample sizes are large

#### **Example: Sparrow bones (cont.)**

Reconsider the data that we analyzed earlier on the humerus bone measurements of a sample of sparrows who survived a severe winter storm and a sample of sparrows who perished in that storm (`bumpus.mtw`). We will use the sample data to construct a 95% confidence interval for  $\mu_S - \mu_P$ .

(a) Examine and comment on comparative boxplots of the humerus lengths between the two groups.

(b) Identify in words what  $\mu_S - \mu_P$  represent in this context.

(c) Use summary statistics from the sample data to construct a 95% confidence interval for  $\mu_S - \mu_P$ .

(d) Comment on the importance of whether or not this interval includes the value zero.

(e) Would a 90% CI for  $\mu_S - \mu_P$  be narrower or wider than the 95% CI? Use Minitab to calculate this interval (Stat > Basic statistics > 2-Sample t; choose "options" to change the confidence level). Comment on whether this interval includes the value zero.

(f) Determine a 95% CI for  $\mu_S - \mu_P$ . How does it compare to the earlier interval? Does this make sense?

**Example: Sleep deprivation**

Researchers studied the effects of sleep deprivation by conducting a study in which some students were randomly assigned to be denied sleep on the first night of the study and other students were randomly assigned to sleep as long as they wanted on that night. All students were allowed unrestricted sleep on the second and third nights of the study. The variable measured was the improvement in score on a visual task between performing the task on the first day and on the fourth day of the study (so a negative score indicates a decrease in performance). The data are in `SleepDeprivation.mtw`.

(a) Examine and comment on comparative boxplots of the score improvements between the two groups.

(b) Identify in words and symbols the parameter being estimated in this context.

(c) Use summary statistics from the sample data to construct a 95% confidence interval for the difference in population means.

(d) Verify your calculation with Minitab (Stat> Basic statistics> 2-sample t).

(e) Comment on the importance of whether or not this interval includes the value zero.

### Study Design:

When drawing conclusions from a study, one of the most important considerations is the manner in which the data were collected.

(a) Identify a key difference in how the sparrows came to be in one of their two groups and in how the students came to be in one of their two groups.

More terminology:

- **Observational Study**- observes individuals and measures variables of interest but does not attempt to influence responses; goal is to *describe* situation.
- **Experiment**- deliberately imposes some treatment on individuals to observe their responses; goal is to study whether treatment *causes* a change in the response.
  - **Response variable**- a variable that measures an outcome or result of a study
  - **Explanatory variable**- a variable that researchers think explains or causes changes in the response variable
- **Lurking variable**- one that has an important effect on the relationship among the variables in a study, but which is not among the explanatory variables studied
- **Confounding**- two variables whose effects on a response variable cannot be distinguished from each other
- **Randomization**- seeks to eliminate confounding variables by producing groups that are similar in all respects before the treatments are applied
- **Causation**- cause/effect conclusions can be drawn from randomized comparative experiments but not from observational studies

(b) Which type of study is the sparrow study? How does that affect the scope of conclusion that can be drawn?

(c) Which type of study is the sleep deprivation study? Identify the explanatory variable and the response variable.

(d) Explain the goal and the importance of random assignment in the sleep deprivation study.

(e) What scope of conclusion is valid with the sleep deprivation study? Explain.

**Example: Smoking during pregnancy**

Many studies reveal that children of mothers who smoke during pregnancy have lower IQ scores than children of mothers who do not smoke during pregnancy.

(a) Identify the explanatory and response variable in this study.

(b) Are these observational or experimental studies? Explain how you know.

(c) Is it valid to conclude that smoking during pregnancy *causes* the lower IQ scores? Explain.

(d) Suggest a potential confounding variable in this study. Explain the possible connection between this variable and the explanatory variable, and also between this variable and the response variable

(e) Describe what conducting an experiment to address this question would entail. Is this feasible? Is it ethical? Explain.