

Stat 218 - Day 17

One-Tailed t -test

Recall from yesterday that we introduced the general idea of a hypothesis test and the specific details of a two-sample t -test:

- **Null hypothesis**, denoted by H_0 , typically a statement of “no effect” or “no difference”
 - In this case: $H_0: \mu_A = \mu_B$
- **Alternative hypothesis**, denoted by H_A , also called the research hypothesis
 - In this case: $H_A: \mu_A \neq \mu_B$
- **Test statistic**, which is a measure of how consistent the observed sample data are with the null hypothesis
 - In this case: $t_s = \frac{(\bar{y}_A - \bar{y}_B) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$
- **P-value**, which is the probability of obtaining such an extreme test statistic if the null hypothesis were true
 - The smaller the P-value, the stronger the evidence against the null hypothesis
 - P-values less than .05 are generally considered to be moderately convincing evidence against H_0 ; P-values less than .01 are generally considered to be strongly convincing evidence against H_0 .
 - In this case: P-value = area in tails of t -distribution beyond $-t_s$ and $+t_s$
- **Significance level**, denoted by α , which is a threshold for determining how small the P-value must be to provide convincing evidence against H_0 .
 - Common choices for α are .10, .05, and .01
- **Test decision**
 - Reject H_0 if P-value $\leq \alpha$ (statistically significant)
 - Fail to reject H_0 if P-value $> \alpha$

Example: Memory Study

We will collect some data from the class and apply this two-sample t -test to the data.

(a) Is this an observational study or an experiment? Explain.

(b) Identify the explanatory and response variables.

(c) State the null and alternative hypotheses, in symbols and in words. (Note that the hypotheses *ca*, and *should*, be stated prior to collecting the data.)

(d) Calculate the value of the test statistic.

(e) Determine the P -value of the test, as accurately as possible from the t -table.

(f) What would the test decision be at the $\alpha=.05$ significance level? Relate this to the context of this study.

(g) How about at the $\alpha=.01$ significance level? Relate this to the context of this study.

(h) Does failing to reject the null hypothesis mean that you accept it? Explain.

(i) How would the test statistic and P -value have changed if the sample sizes were larger and all else remained the same? Explain.

What's wrong with this analysis? Just that it fails to incorporate one piece of information: I conjectured *before* collecting the data that the "JFK" group would do better on average than the "JFKC" group. So, the research (alternative) hypothesis should be *directional*. The only change in the procedure is that we calculate the P -value as the one-tailed area beyond the value of the test statistic t_s , assuming that the data deviate from H_0 in the direction specified by H_A .

(j) Restate the hypotheses with the directional alternative.

(k) Do the sample data deviate from H_0 in the direction specified by H_A ? In other words, is the sample mean higher in the "JFK" group than in the "JFKC" group?

(l) If the answer to the previous question is "yes," calculate the one-tailed P -value. How does it compare to the original P -value?

(m) What conclusion would you reach at the $\alpha=.05$ significance level?

(n) Use Minitab to re-create this analysis (Stat> Basic Statistics> 2-Sample t, click "options" to specify the alternative).

Example: Fish Oil Diet (cont.)

(a) Is it appropriate to use a directional alternative to analyze the results of this study? Explain.

(b) What effect does the directional alternative have on the analysis and conclusions? Explain.