Stat 218 - Day 21
Chi-Square Goodness of Fit Tests

Today and for the next several days, we turn our attention to the analysis of *categorical* data.

Example: Mendel pea experiments
In Mendel's classic pea experiments to test his genetic theory, he predicted the following proportional breakdown for four types of peas:

round and yellow	wrinkled and yellow	round and green	wrinkled and green
9/16	3/16	3/16	1/16

Consider the following experimental data on 100 such peas :

59	19	14	8

How do we test whether the experimental data support or refute the genetic theory? The basic idea is to

- Calculate the **expected** counts given the (null) hypothesis that the theory is correct
- Compare the **observed** counts to these expected counts
- Construct a test statistic that measures the **discrepancy** between them
- Determine the probability of getting such an extreme discrepancy if the theory were correct (*P*-value)
- Reject the theory if this *P*-value is small

This procedure is called a **chi-square test of goodness-of-fit**.

(a) Calculate the expected counts for this study, under the hypothesis that the theory is correct.

The test statistic is denoted by χ_s^2 and is calculated as: $\chi_s^2 = \sum \frac{(O-E)^2}{E}$.

(b) Calculate the value of the test statistic for these data.

The *P*-value is found as the upper tail area of the chi-square distribution (Table 9) with degrees of freedom equal to the number of categories minus one.

(c) Determine the *P*-value of this test.

(d) Write out the null and alternative hypotheses. Would you reject the null hypothesis at the α=.10 level?

(e) Summarize your conclusion about whether the experimental data support or refute the genetic theory, and explain how your conclusion follows from this test.

Example: Birthdays of week

Last quarter I asked my students what day of the week they were born on. The sample results were:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Total
24	15	15	26	20	18	20	138

Conduct a chi-square test of whether these sample data provide evidence that Cal Poly students are not *equally likely* to be born on the seven days of the week.

Binary data: We can apply these techniques to categorical data that are binary (also called dichotomous), meaning that only two categories are possible. The procedure is exactly the same, but directional hypotheses are possible. With a directional hypothesis, if the sample data fall in the hypothesized direction, calculate the P-value by dividing the tail area from the chi-square distribution by two.

Example: Halloween treats
Stemming from concern over the nation's obesity epidemic, researchers investigated whether children might be as tempted by toys as by candy for Halloween treats. Test households in five Connecticut neighborhoods offered children two bowls: one with lollipops or fruit candy and one containing small, inexpensive Halloween toys, like plastic bugs that glow in the dark. Of the 283 children from 3 to 14 whose reactions were tallied, 148 chose candy and 135 chose toys.

Conduct a chi-square test of whether children in the population are equally likely to choose either type of treat. As always:

- Report the null and alternative hypotheses
- Calculate the test statistic and P-value
- State your conclusion in context

Example: Transplant mortality
Researchers examined 371 heart transplant operations at a hospital that was suspected of having an unusually high mortality rate. The national benchmark for comparison was 15%. The researchers found that 79 patients in this hospital died within 30 days of the transplant operation. Test whether these the sample data suggest that the probability of death within 30 days of the transplant operation in this hospital exceeds the national benchmark of 15%. As always:

- Report the null and alternative hypotheses
- Calculate the test statistic and P-value
- State your conclusion in context