

Stat 218 - Day 24

More on 2×2 Tables: Difference in Proportions, Relative Risk, Odds Ratio

Today we continue analyzing 2×2 tables. We will consider three ways to measure the difference in the response variable distribution between the two categories of the explanatory variable:

- Difference in proportions (probabilities)
- Relative risk
- Odds ratio

Example: AZT and pregnancy (cont.)

Recall that we analyzed the results of an experiment that resulted in the following 2 × 2 table:

	Pregnant woman received placebo	Pregnant woman received AZT	Total
HIV + baby	40	13	53
HIV – baby	120	151	271
Total	160	164	324

We calculated earlier that .250 of the women in the placebo group had an HIV + baby, compared to .079 in the AZT group.

The **difference in proportions** is self-explanatory.

(a) Calculate the difference between these proportions.

We can calculate a confidence interval for the difference in population proportions between two

groups by: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$

(b) Construct a 95% confidence interval for the difference $p_{\text{plac}} - p_{\text{AZT}}$.

(c) Interpret this interval. Pay particular attention to whether it includes zero or is entirely positive or entirely negative.

The **relative risk** is the ratio of the two group's proportions.

(d) Calculate the relative risk of having an HIV+ baby between the placebo group and the AZT group.

The interpretation here is that a woman receiving placebo is 3.15 times more likely to have an HIV+ baby than if she takes AZT.

To study the odds ratio, we first must consider **odds**. The odds of an event is the ratio of the probability that the event happens to the probability that it does not happen.

(e) Consider a woman receiving a placebo. Calculate the odds that she has an HIV+ baby.

The **odds ratio** is the ratio of the odds between the two groups.

(f) Calculate the odds ratio of having an HIV+ baby between the placebo group and the AZT group.

A shortcut for calculating the odds ratio is to multiply along the two diagonals in the table and then take the ratio of those two products. The convention is to put the larger ratio in the numerator, to force the odds ratio to be at least one.

(g) Verify that this shortcut produces the same odds ratio that you calculated in (f).

To construct a confidence interval for a population odds ratio, denoted by θ :

- First calculate the sample odds ratio, denoted by $\hat{\theta}$.
- Next take the natural log of the sample odds ratio.
- Then find a confidence interval for $\log(\theta)$ by using: $\log(\hat{\theta}) \pm z * \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}$.
- Finally, exponentiate the endpoints to get a CI for θ rather than for $\log(\theta)$.

(h) Determine a 95% confidence interval for the population odds ratio of having an HIV+ baby between the placebo group and the AZT group.

Since the odds ratio is harder to interpret and to work with than the difference in proportions or the relative risk, why do we bother with it? Because the odds ratio makes sense no matter how the data were collected, whether the row totals were chosen in advance or the column totals were chosen in advance or neither set of totals were chosen in advance.

Example: Sleepless Drivers

New Zealand researchers collected sleep data on a sample of drivers who had been in a car crash and on an independent sample of drivers who had not been in a car crash. They found that 61 of the 535 crash drivers had not gotten a full night's sleep in the previous week, compared to 44 of the 588 control drivers.

(a) Organize these data into a 2×2 table.

(b) What proportion of these drivers were involved in a car crash?

(c) Does it make sense to use this proportion to estimate the proportion of all New Zealand drivers who were involved in a car crash that week? Explain.

(d) Does it make sense to calculate the odds ratio of not having a full night's sleep between these two groups? If so, calculate and interpret it.