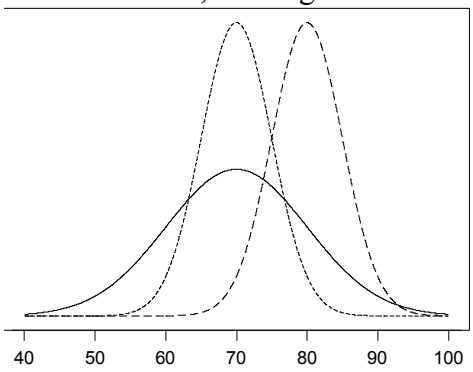
Stat 218 - Day 7
Normal distributions

Many phenomena, especially physical measurements, follow a bell-shaped curve. A variable that follows a symmetric, bell-shaped curve is said to have a **normal distribution**. A normal distribution is characterized by two values: its mean (μ) and standard deviation (σ).

- The mean μ indicates where the center and peak of the distribution are.
- The standard deviation σ is the distance from the mean to where the curvature changes.

Example: For the three normal curves below, take a guess for its mean and standard deviation.



The total area under a normal curve is one. The area under the curve over a certain region indicates:

- The proportion of values in that region
- The probability that a randomly selected observational unit will be in that region

To find the area under the curve over a certain interval, we:

- Standardize by subtracting the mean and dividing by the standard deviation (i.e., find a *z*-score)
- Use the table of standard (mean 0, std dev 1) normal probabilities, which gives the area under the curve to the left of that value

- Or you can use the Java applet called "normal probability calculations"
- Or you can use Minitab (`Calc> Probability distributions> Normal; cumulative probability`)

Advice: Always draw a sketch of the normal distribution, and shade the area corresponding to the probability/proportion of interest.

Example: SAT and ACT (cont.)
Recall that scores on the SAT exam are normally distributed with mean 1000 and std dev 180, while scores on the ACT exam are normally distributed with mean 21 and std dev 6.
(a) Draw sketches of these two distributions. Label the scales as accurately as possible.

(b) Suppose that Bobby scores 1180 on the ACT and Kathy scores 30 on the ACT. Determine what percentage of students score below each of these scores on the respective exams. Is your answer for Bobby consistent with what the empirical rule revealed?

(c) Suppose that Peter scores 910 on the SAT and Kelly scores 15 on the ACT. Determine what percentage of students score below each of these scores on the respective exams. Is your answer for Kelly consistent with what the empirical rule revealed?

(d) About what score is needed on the SAT to be at the 90^{th} percentile (the value so that 90% score below that value)? How about on the ACT? [*Hints*: Start with a sketch. You will need to read the table "in reverse," looking up the area in the middle of the table and reading backwards to find the relevant z-score. Then you will have to un-convert the z-score back to the SAT/ACT scale.]

Example: Birthweights
Birthweights of babies in the United States can be modeled by a normal distribution with mean 3250 grams and standard deviation 550 grams.
(a) Draw a sketch of this distribution.

(b) About what percentage of babies are considered of low birth weight (less than 2500 grams).

(c) About what percentage of babies weigh more than 10 pounds (4536 grams) at birth?

(d) Describe two different ways that you could have used the table to answer the previous question.

(e) Determine the probability that a randomly selected baby weighs between 3000 and 4000 grams at birth. [*Hint*: Decide what to do with the tabled values for the two relevant z-scores.]

(f) How little would a baby have to weigh to be among the lightest 2.5% of all newborns?

(g) How much would a baby have to weigh to be among the heaviest 10% of all newborns?