1. (6 pts) Suppose that you are considering whether to invest in a new business that a friend of yours is starting. After researching the possibilities, you decide that you will gain $50,000 if the business is successful, and you will lose $20,000 if the business is not successful.

a) (2 pts) Suppose that 30% of all such businesses are successful, so you consider .3 to be the probability that your friend’s business will be successful. Determine the expected value of your net profit.

b) (2 pts) Write a sentence interpreting what this expected value means.

c) (2 pts) Now ignore the 30% figure and instead let $p$ represent the probability that your friend’s business will be successful. Determine the values of $p$ for which the expected value of your net profit is positive.

2. (8 pts) Suppose that among the pre-owned cars available in a large dealership, 80% have air conditioning, 70% have a CD player, and 65% have both.

a) (2 pts) Determine the proportion of these cars that have neither air conditioning nor a CD player. (As always, show your method of solution.

b) (2 pts) Determine the conditional probability that a randomly selected car has a CD player, given that it has air conditioning.

c) (2 pts) Are the events \{randomly selected car has air conditioning\} and \{randomly selected car has a CD player\} independent? Justify your answer.

d) (2 pts) Are the events \{randomly selected car has air conditioning\} and \{randomly selected car has a CD player\} disjoint? Explain briefly.

3. (11 pts) A teacher conducted a study to see whether students could distinguish between the tastes of two brands of cola. Each of 30 students was presented with three cups: Two cups contained the same brand of cola, and the third cup contained a different brand of cola. Each student was asked to identify which cup contained the different brand. Let the random variable $X$ represent the number of students who correctly identify the different cup.

a) (1 pt) If students are really not able to distinguish, and so they randomly choose among the three cups, what would be the probability of correctly identifying the different one?

b) (3 pts) It turned out that 16 of 30 students correctly identified the different cup. Fill in the following blanks to specify how to calculate the p-value for this study:

$$ p\text{-value} = P(X \text{ ________}), \text{ where } X \text{ has a } \text{distribution} $$

with parameter values ________________________________________________.
c) (3 pts) The p-value turns out to equal .019. Finish the following sentence to interpret this p-value:

The probability is .019 that ____________________________________________,
assuming that ____________________________________________.

d) (4 pts) Based on this p-value, would you conclude that the sample data provide fairly strong evidence that students really can identify the different cola more often than by random chance? Also explain the reasoning process behind your answer.

4. (9 pts) Suppose that when the interest rate goes up in a particular month, stock prices go down with probability .8 and stock prices go up with probability .2. Also suppose that when the interest rate goes down in a given month, stock prices go down with probability .3 and stock prices go up with probability .7. Finally, suppose that the probability is .4 that the interest rate will go up next month and .6 that the interest rate will go down next month.

Use the following symbols for these events:
   SU = {stock price goes up}  SD = {stock price goes down}
   IU = {interest rate goes up}  ID = {interest rate goes down}

a) (3 pts) Express the following three probabilities from the paragraph above in terms of these event symbols:
   .8 =
   .3 =
   .4 =

b) (3 pts) Determine the probability that the interest rate goes down and stock prices go up next month. Justify your answer with a probability tree or a probability table or a probability rule.

c) (3 pts) Determine the probability that stock prices go up next month.

5. (5 pts) We showed in class that in a room of 30 people, the probability is .706 that at least two people have the same birthday. Now consider (but do not bother to calculate) the probability that in a room of 30 people, at least one person was born on my birthday, March 15.

a) (2 pts) Do you think this probability is smaller than .706, larger than .706, or equal to .706? Give an intuitive explanation, but do not bother to calculate this probability.

b) (3 pts) Write an expression that indicates how to calculate this probability, but do not bother to calculate its numerical value.
6. (5 pts) Suppose that a manager must choose 3 employees to be laid off from among the 10 employees that she supervises, and these 10 employees include 4 who are over age 50. If the decision about which employees to lay off is made completely at random, what is the probability that all 3 of the employees selected are over age 50?

7. (6 pts) Suppose that Jose has applied for a job with three different companies. He thinks that he has a 40% chance of getting a job offer from company A, a 60% chance of getting a job offer from company B, and a 90% of getting a job offer from company C. Furthermore, he believes that whether or not he receives a job offer from any company is independent of whether or not he receives a job offer from any other company

   a) (3 pts) What is the probability that Jose receives a job offer from all three companies?

   b) (3 pts) What is the probability that Jose receives a job offer from at least one of these three companies?