STAT 252 Handout 9 Winter 2010
Chi-Square Tests for Two-Way Tables

Today we continue our study of categorical data with more than two possible categories. Now we consider studies with *two* such variables, for which the data can be organized in a two-way table.

Example 1: Pursuit of Happiness
Have Americans gotten happier or less happy over the past few generations? Or has the happiness level remained fairly constant? The General Social Survey (GSS) interviews a random sample of adult Americans every two years, and one of the questions asks respondents to rate their general happiness level. Sample results for 1972, 1988, and 2004 are summarized in the following 2×3 table:

	1972	1988	2004	Total
Very Happy	486	498	419	1403
Less Than Very Happy	1120	968	918	3006
Total	1606	1466	1337	4409

a) Identify the observational units in this study.

b) Identify and classify the explanatory and response variables in this study.

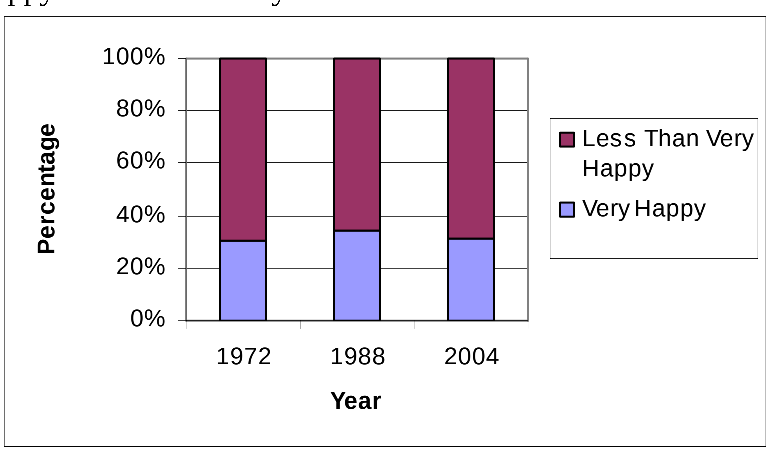
Explanatory: Type:

Response: Type:

c) Is this an observational study or an experiment? Explain.

d) What kind of graphical display is appropriate for these data?

e) Comment on what the following graph reveals about the sample proportions who considered themselves very happy in each of these years.



f) Use appropriate symbols to state the null hypothesis that the population proportion of adult Americans who considered themselves to be very happy was the same in these three years.

g) For the three years combined, what proportion of respondents were very happy?

h) If this same proportion (your answer to g) of the 1606 respondents in the year 1972 had been very happy, how many people would this have been? Record your answer with 2 decimal places.

i) Repeat (h) for the years 1988 and 2004.

1988: 2004:

You have calculated the **expected counts** under the null hypothesis that the three years have the same population proportion of very happy people. A more general technique for calculating the expected count of the cell in row i and column j of the table is to take the marginal total for row i times the marginal total for column j divided by the grand total:

$$E_{ij} = \frac{R_i C_j}{n} = \frac{\text{(row total)(column total)}}{\text{grand total}}$$

j) Use this more general formula to calculate the expected count of "less than very happy" people in the year 1988. Record this in parentheses in the bottom middle cell of the table below:

	1972	1988	2004	Total
Very Happy	486 (511.05)	498 (466.50)	419 (425.45)	1403
Less Than Very Happy	1120 (1094.95)	968 ( )	918 (911.55)	3006
Total	1606	1466	1337	4409

Now our task is to use a test statistic to measure how far the observed counts fall from the expected counts. We will use the same calculation that we did with the goodness-of-fit test:

$$X^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \sum_{\text{all cells}} \frac{(\text{observed - expected})^2}{\text{expected}}.$$

k) Calculate the value of $\frac{(O-E)^2}{E}$ for the less than very happy people in 1988 (i.e., for the bottom, middle cell of the table).

l) The other contributions to the test statistic calculation are provided below. Insert your answer to (k) and sum these to calculate the value of the test statistic.

X^2 = 1.228 + 2.127 + 0.098
+ 0.573 + ________ + 0.046
=

m) What kind of values (e.g., large or small) of the test statistic constitute evidence against the null hypothesis that the three populations have the same proportions? Explain.

Once again it turns out that under the null hypothesis of equal population proportions, the test statistic X^2 has a chi-square distribution. The degrees of freedom is now equal to (r–1)×(c–1), number of rows times number of columns. This test is considered valid as long as all of the expected counts are at least five. Since large values of the test statistic provide evidence against the null hypothesis, the p-value for the chi-square test is the probability of exceeding the value of the test statistic. As always, the smaller the p-value, the stronger the evidence that there is a difference in the population proportions among the populations.

n) Use the chi-square table to determine (as accurately as possible) the *p*-value for these sample data.

o) Enter the observed counts into Minitab, and use Minitab to calculate the expected counts, test statistic, and p-value (`Stat> Tables> Chi-Square test (Two-Way Table in Worksheet`). Do Minitab's results agree with yours?

p) Would these sample data have been very unlikely to occur by chance alone if the population proportions of very happy people had been identical for these three years? What test decision would you reach at the $\alpha = .05$ significance level?

q) Summarize the conclusion that you draw from this study.

Summary of chi-square test for two-way table:

- The null hypothesis says that the distribution of the row variable is the same for every category of the column variable, or (equivalently) that the two variables are independent.
- Expected counts are calculated as: $E = \dfrac{(\text{row total}) \times (\text{column total})}{\text{grand total}}$
- The test statistic is calculated as: $X^2 = \sum \dfrac{(O - E)^2}{E}$
- The *p*-value is again found from the chi-square distribution, with degrees of freedom equal to $(r\text{-}1)\times(c\text{-}1)$.
- The technical conditions are that you have a random sample or random assignment, and that all of the expected counts are at least five.

Example 2: Newspaper reading habits

A sample of 811 men and 1059 women were asked how often they read a newspaper. The results are summarized in the table:

	Men	Women	Total
Every day	375 ( )	430 ( )	805
A few times per week	182 ( )	238 ( )	420
Once per week	121 ( )	173 ( )	294
Less than once per week	133 ( )	218 ( )	351
Total	811	1059	1870

a) Identify the two categorical variables in this study. What are the values of r and c for this $r\times c$ table?

b) State the null and alternative hypotheses for testing whether these sample data provide evidence that newspaper reading frequency differs between men and women in the population.

c) Enter the observed counts into Minitab, and use Minitab to calculate expected counts, the X^2 test statistic, and *p*-value. Record the expected counts in parentheses of the table above.

d) Do the distributions of newspaper reading frequency differ significantly between men and women at the α = .05 significance level? How about at the α = .01 level?

e) Look at the eight pieces of the calculation of the X^2 test statistic. Which cells of the table do the two largest pieces come from? Are the observed counts above or below the expected counts in those cells?

f) Summarize your conclusions about whether men and women differ in their newspaper reading habits.

Example 3: Federal spending on space program
The two-way table below presents a summary of responses to a question on the 2004 General Social Survey about political inclination and opinion regarding how much the federal government spends on the space program:

	Liberal	Moderate	Conservative
Too little	23	25	26
About right	75	109	109
Too much	53	120	92

a) Identify the two categorical variables in this study.

b) What are the values of r and c for this $r \times c$ table?

In the "newspaper reading" example, researchers took one random sample of men and an independent random sample of women, classifying the people according to their newspaper reading frequency. But in this "government spending" example, researchers took one random sample of people and classified them according to two variables: political viewpoint and opinion about federal spending on the space program. The chi-square test in this context tests the null hypotheses that these two variables are **independent**.

c) Conduct the chi-square test. (Feel free to use Minitab.) Report the hypotheses, test statistic, and *p*-value. Also state your conclusion in context and explain the reasoning process by which it follows from your analysis.