STAT 301 Review of Inference for One Proportion

When my tennis racquet is spun, is it equally likely to land with its label facing up or down? Or does the spinning process favor one outcome more than the other? I once investigated this question by spinning my tennis racquet many times. For each spin I recorded whether the racquet landed with the label up or down.

a) Describe (in words) the relevant parameter whose value is being investigated with this study.

b) Write the appropriate null and alternative hypotheses (in symbols).

I spun my racquet 100 times, finding that it landed with the label up in 46 of those spins.

c) Describe how you could use a coin to conduct a simulation analysis of whether this result constitutes strong evidence that my racquet spinning process is not equally likely to land with its label facing up or down. Provide enough detail that someone else could implement the simulation and draw the appropriate conclusion.

d) Use the coin-tossing applet (which you can find under the notes for day1) to simulate 1000 repetitions of 100 spins each. Use the simulation result to produce an approximate p-value. Include a copy (or rough sketch) of the applet results in your report.

e) Use the binomial distribution to calculate the p-value exactly. (Be sure to indicate how you calculate this probability: what values you use for n and π, and what region you find the probability of.)

f) Check whether the normal approximation (Central Limit Theorem) is valid here.

g) Describe what the CLT says about the (approximate) sampling distribution of the sample proportion $\hat{p}$, assuming that the null hypothesis is true. Be sure to refer to shape, center, and spread.

h) Calculate the test statistic by finding the z-score for the observed sample proportion $\hat{p}$.

i) Determine the (approximate) p-value from the standard normal distribution.

j) What test decision would you make at the .05 significance level?

k) Do the conditions for the (Wald) normal-based confidence interval hold here?

l) Produce a 95% confidence interval for the parameter, using the Wald procedure if the conditions are met but using the adjusted Wald procedure if they are not met.

m) Is the confidence interval consistent with the test decision? Explain.

n) Summarize your conclusion about the original question that motivated this study.

o) Use the binomial distribution to determine the rejection region (in terms of number of "up" results in the sample) for the .05 significance level.

p) Use the binomial distribution to determine the power of this test, using the .05 significance level, when the actual probability of my spun tennis racquet landing "up" is .6.

q) Use the normal approximation to determine the rejection region (in terms of the sample proportion $\hat{p}$) for the .05 significance level.

r) Use the normal approximation to determine the power of this test, using the .05 significance level, when the actual probability of my spun tennis racquet landing "up" is .6.

s) Use the normal approximation to determine how large the sample size *n* needs to be in order for the 95% confidence interval to have margin-of-error $\leq$ .08.

Solution

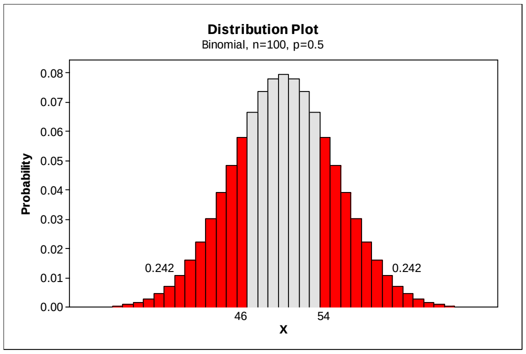
a) The parameter is the probability that my spun tennis racquet lands with the label up, denoted by π. This is equivalent to the long-run proportion of times that the racquet would land up if I were to spin it indefinitely.

b) H_0: $\pi = .5$ vs H_a: $\pi \neq .5$

c) Toss a fair coin 100 times, counting the number of tosses that land heads, representing a racquet spin that lands up. Repeat this process of 100 coin flips a large number of times (say, 1000), each time counting the number of heads. Then determine the proportion of those 1000 repetitions (of 100 slips each) that produced either 46 or fewer heads or 54 or more heads. If this proportion is very small (say, less than .05), that indicates that a result as extreme as the one observed would rarely happen with a 50/50 process, and so in that case we would conclude that the racquet really does favor one side or the other. But if this proportion is not very small (say, greater than .05), that indicates that a result as extreme as the one observed is fairly consistent with a 50/50 process, and so in that case we would not reject the hypothesis that the racquet spinning is a 50/50 process.

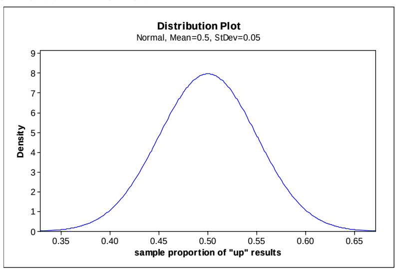
d) My approximate p-value is (253+204)/1000 = .457, the proportion of the 1000 simulated samples (of 100 spins each) that resulted in 46 or fewer heads or 54 or more heads.

e) This p-value is $\Pr(X \leq 46) + \Pr(X \geq 54)$, where X has a binomial distribution with $n = 100$ and $\pi = .5$. Minitab reveals this probability to be .484, as shown below:



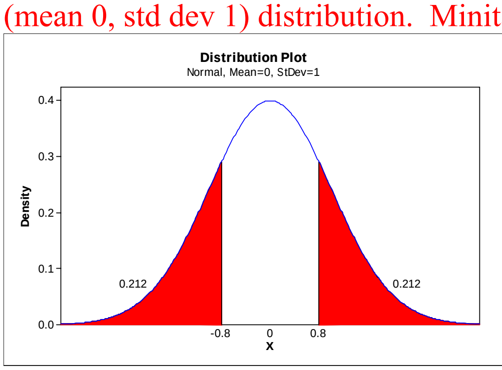
f) Yes, because $n \times \pi_0 = 100(.5) = 50$ is larger than 10, as is $n \times (1-\pi_0) = 100(.5) = 50$.

g) The CLT says that the sample proportion $\hat{p}$ will vary according to an approximate normal distribution with mean .5 and standard deviation $\sqrt{\frac{.5 \times .5}{100}} = .05$. A sketch of this distribution is shown here:



h) $z = \frac{.46 - .5}{.05} = -0.8$

i) The p-value is approximately $\Pr(Z \leq -0.8) + \Pr(Z \geq 0.8)$, where Z denotes a standard normal (mean 0, std dev 1) distribution. Minitab reveals this probability to be .424, as shown here:



j) Fail to reject H_0, because the p-value is larger than .05. We have little/no evidence to doubt that the racquet lands up 50% of the time.

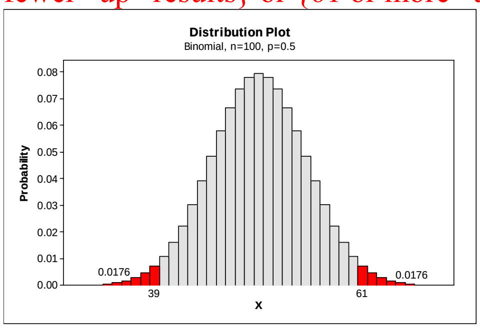
k) Yes, because $n \times \hat{p} = 100(.46) = 46$ is larger than 10, as is $n \times (1-\hat{p}) = 100(.54) = 54$.

l) A 95% confidence interval for π is: $.46 \pm 1.96\sqrt{\frac{.46 \times .54}{100}}$, which is $.46 \pm 1.96(.049)$, which is $.46 \pm .098$, which is (.362, .558). We can be 95% confident that between 36.2% and 55.8% of all spins with my racquet would land with the label up.

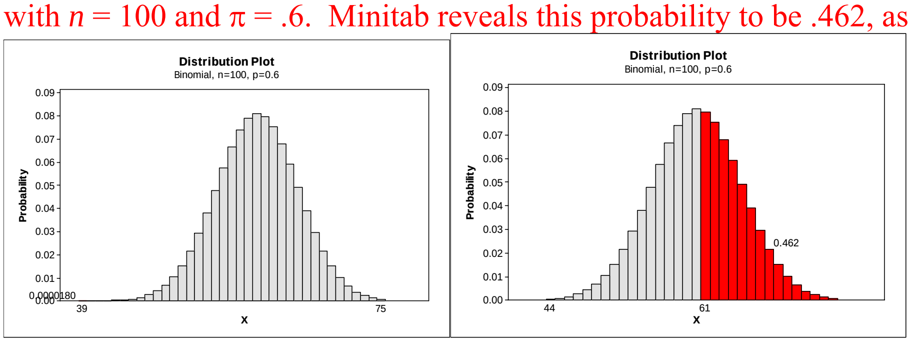
m) Yes, we did not reject the hypothesis that $\pi = .5$ at the $\alpha = .05$ level, and .5 appears within the 95% confidence interval for π.

n) The sample data provide no reason to doubt that my racquet lands "up" 50% of the time.

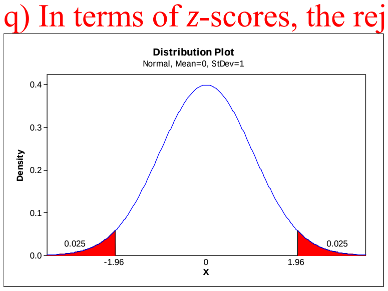
o) The rejection region is found from a binomial distribution with $n = 100$ and $\pi = .5$, looking for a two-tailed region with probability .05 or less. Minitab reveals the rejection region to be {39 or fewer "up" results} or {61 or more "up" results}, as seen here:



p) Power is the probability of rejecting H_0: $\pi = .5$, in this case when the actual probability is $\pi = .6$. This probability (power) is $Pr(X \leq 39) + Pr(X \geq 61)$, where X has a binomial distribution with $n = 100$ and $\pi = .6$. Minitab reveals this probability to be .462, as shown below:

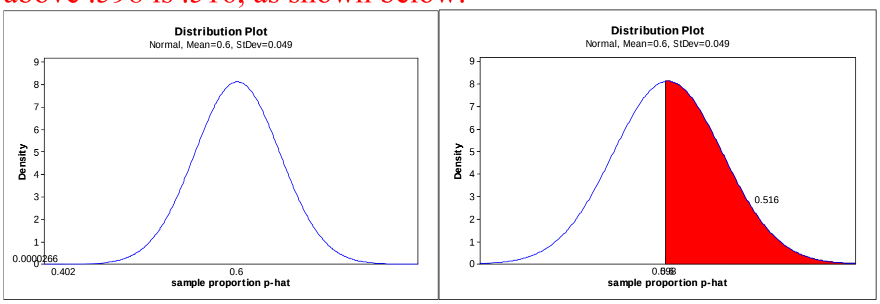


q) In terms of *z*-scores, the rejection region is $Z \leq -1.96$ or $Z \geq 1.96$, as shown here:



In terms of the sample proportion $\hat{p}$, the *z*-score is $z = \dfrac{\hat{p} - .5}{\sqrt{\dfrac{.5 \times .5}{100}}}$. Solving the rejection region inequalities for $\hat{p}$ gives: $\hat{p} \le .5 - 1.96\sqrt{\dfrac{.5 \times .5}{100}}$ or $\hat{p} \ge .5 + 1.96\sqrt{\dfrac{.5 \times .5}{100}}$, which is $\hat{p} \le .402$ or $\hat{p} \ge .598$.

r) When $\pi = .6$, the sampling distribution of $\hat{p}$ is approximately normal with mean .6 and standard deviation $\sqrt{\dfrac{.6 \times .4}{100}} \approx .049$. In this case, the probability that $\hat{p}$ will be below .402 or above .598 is .516, as shown below:



s) Using the observed sample proportion .46 as an estimate for $\hat{p}$, we need to solve $1.96\sqrt{\dfrac{.46 \times .54}{n}} \le .08$. Solving gives $n \ge \dfrac{(1.96)^2 \times .46 \times .54}{(.08)^2} \approx 149.1$, so 150 spins would be needed.