

STAT 301 Review of Inference for One Proportion

When my tennis racquet is spun, is it equally likely to land with its label facing up or down? Or does the spinning process favor one outcome more than the other? I once investigated this question by spinning my tennis racquet many times. For each spin I recorded whether the racquet landed with the label up or down.

- a) Describe (in words) the relevant parameter whose value is being investigated with this study.
- b) Write the appropriate null and alternative hypotheses (in symbols).

I spun my racquet 100 times, finding that it landed with the label up in 46 of those spins.

c) Describe how you could use a coin to conduct a simulation analysis of whether this result constitutes strong evidence that my racquet spinning process is not equally likely to land with its label facing up or down. Provide enough detail that someone else could implement the simulation and draw the appropriate conclusion.

d) Use the coin-tossing applet (which you can find under the notes for day1) to simulate 1000 repetitions of 100 spins each. Use the simulation result to produce an approximate p-value. Include a copy (or rough sketch) of the applet results in your report.

e) Use the binomial distribution to calculate the p-value exactly. (Be sure to indicate how you calculate this probability: what values you use for n and π , and what region you find the probability of.)

f) Check whether the normal approximation (Central Limit Theorem) is valid here.

g) Describe what the CLT says about the (approximate) sampling distribution of the sample proportion \hat{p} , assuming that the null hypothesis is true. Be sure to refer to shape, center, and spread.

h) Calculate the test statistic by finding the z-score for the observed sample proportion \hat{p} .

i) Determine the (approximate) p-value from the standard normal distribution.

j) What test decision would you make at the .05 significance level?

k) Do the conditions for the (Wald) normal-based confidence interval hold here?

l) Produce a 95% confidence interval for the parameter, using the Wald procedure if the conditions are met but using the adjusted Wald procedure if they are not met.

m) Is the confidence interval consistent with the test decision? Explain.

n) Summarize your conclusion about the original question that motivated this study.

- o) Use the binomial distribution to determine the rejection region (in terms of number of “up” results in the sample) for the .05 significance level.
- p) Use the binomial distribution to determine the power of this test, using the .05 significance level, when the actual probability of my spun tennis racquet landing “up” is .6.
- q) Use the normal approximation to determine the rejection region (in terms of the sample proportion \hat{p}) for the .05 significance level.
- r) Use the normal approximation to determine the power of this test, using the .05 significance level, when the actual probability of my spun tennis racquet landing “up” is .6.
- s) Use the normal approximation to determine how large the sample size n needs to be in order for the 95% confidence interval to have margin-of-error $\leq .08$.