

You may work with one partner on this assignment, submitting one report with both names, provided that both students contribute substantially to the work. Word-processed reports are preferred to hand-written ones. Please copy/paste relevant computer output into your report as appropriate.

Pet CPR?

A national survey conducted on October 1-5, 2009 asked pet owners whether they would perform CPR on their pet in the event of a medical emergency. In the sample of 1116 pet owners, 58% said that they are at least somewhat likely to perform CPR on their pet.

a) Use this information to determine a 90% confidence interval for the proportion of all American pet owners who are at least somewhat likely to perform CPR on their pet. Be sure to check technical conditions for the procedure and also interpret what the interval reveals.

This 90% confidence interval is: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, which is $.58 \pm 1.645 \sqrt{\frac{.58 \times .42}{1116}}$, which is $.58 \pm 1.645 \times .0148$, which is $.58 \pm .024$, which is $(.556, .604)$. We can be 90% confident that between 55.6% and 60.4% of all American pet owners would say that they are at least somewhat likely to perform CPR on their pet. This procedure is valid because $n\hat{p} = 1116(.58) \approx 647$ and $n(1-\hat{p}) = 1116(.42) \approx 469$ are both greater than 10.

b) State the appropriate null and alternative hypotheses, in symbols and in words, for testing whether the proportion of dog owners who would perform CPR on their pet is different from the proportion of cat owners who would perform CPR on their pet.

The null hypothesis says that the proportion of all American dog owners who would perform CPR on their pet is equal to that of all American cat owners ($H_0: \pi_{\text{dog}} = \pi_{\text{cat}}$). The alternative hypothesis says that these proportions are different ($H_a: \pi_{\text{dog}} \neq \pi_{\text{cat}}$).

The article that I read reports that 63% of dog owners and 53% of cat owners in the sample said that they would perform CPR on their pet. But the article does not reveal the numbers of dog and cat owners in the sample. Suppose for now that there were 500 dog owners and 500 cat owners.

c) Verify that the technical conditions for a z-test are satisfied.

We'll have to assume that the samples were chosen randomly from the population of all American dog owners and all American cat owners, because we aren't told how the sample was chosen. The other condition is satisfied because $.63(500) = 315$, $.37(500) = 185$, $.53(500) = 265$, and $.47(500) = 235$ are all larger than 5.

d) Calculate the test statistic and p-value of the test. (Feel free to use Minitab.) Also indicate the smallest significance level at which you would reject the null hypothesis, and summarize your conclusion from this test.

Minitab reveals that the test statistic is $z = 3.20$ and p-value is .001, as seen in the output:
Test for difference = 0 (vs not = 0): $Z = 3.20$ P-Value = 0.001
The smallest significance level at which the null hypothesis would be rejected is .001.

The very small p-value indicates very strong evidence that the proportion of dog owners who say that they would perform CPR on their pets does indeed differ from the analogous proportion for cat owners.

e) Determine a 90% confidence interval for the difference in population proportions of dog vs. cat owners who would perform CPR on their pet. Also write a sentence interpreting what this interval reveals.

Minitab reports the following confidence interval for $\pi_{\text{dog}} - \pi_{\text{cat}}$:
90% CI for difference: (0.0489194, 0.151081)
We can be 90% confident that the proportion of all American dog owners who would perform CPR on their pet is higher than that of all American cat owners by between 4.9 and 15.1 percentage points.

f) Is your test conclusion consistent with the confidence interval? Explain.

Yes, the test and CI give consistent results. The test rejects that the proportions are equal (at the .10 level), and the 90% CI for the difference in proportions does not include 0.

Now suppose that the sample sizes had been only 100 dog owners and 100 cat owners, with the same sample proportions of .63 and .53, respectively, answering that they would perform CPR on their pet.

g) Repeat questions d) and e) for this scenario.

Minitab reports the following output:
90% CI for difference: (-0.0133637, 0.213364)
Test for difference = 0 (vs not = 0): $Z = 1.44$ P-Value = 0.149
In this case (with smaller sample sizes) the difference between the sample proportions (.65 and .55) is not statistically significant at any reasonable significance level, because the p-value is .149. In fact, the smallest significance level at which the null hypothesis would be rejected is .149. The 90% confidence interval for $\pi_{\text{dog}} - \pi_{\text{cat}}$ does include the value 0, suggesting that it's plausible that the two population proportions are the same between dog and cat owners. The interval also reveals that the proportion of dog owners would give CPR could be as much as .213 larger than that of cats or as much as .013 less than that of cats.

h) Summarize how the change in sample size affected the significance test and confidence interval. Also explain why it makes sense that these changed as they did.

Decreasing the sample size produces a *less* significant result, with a smaller test statistic and larger p-value. In other words, the smaller sample size produces less evidence that the population proportions actually differ. The confidence interval is wider, to the point of including negative values as well as positive ones, with the smaller sample size.