HW 4: Playing Word Games (assigned on Wed Oct 1; due on Mon Oct 6)
Topics: Significance level, rejection region, type I error, type II error, power, factors that influence power
Computer use: Power simulation applet, R (for binomial probability calculations)

You may work with one partner on this assignment, handing in one report with both names, provided that both of you contribute substantially to the work. Word-processed reports are much preferred to hand-written ones. Please copy/paste relevant, well-labeled computer output into a Word file as appropriate.

I think I’m better than my friend at the game Words with Friends, so I claim that the probability that I win in any one game that we play is greater than one-half.

a) State the appropriate null and alternative hypotheses for testing my claim, using appropriate symbols.

b) Describe (in words) what Type I error means in this situation.

c) Describe (in words) what Type II error means in this situation.

d) Describe (in words) what power means in this situation.

In the 120 games that we have played, I have won 67 and she has won 53.

e) Use these data to determine an approximate p-value through simulation and an exact binomial p-value for testing the hypotheses you stated in a).

f) Would you reject the null hypothesis at the .05 significance level? Explain why or why not.

g) Do the observed data (67 wins for me, compared to 53 wins for my friend) provide convincing evidence that I am better at the game than my friend? Do the data provide convincing evidence that I am not better at the game than my friend? Explain.

For the remaining questions, you may use either the Power Simulation applet to produce approximate probabilities or R to produce exact probability calculations. (Include screen captures of applet/R output with your answers.)

h) Determine the rejection region for this test, using the \( \alpha = .05 \) significance level. In other words, determine how many games I would have to win in a sample of 120 games

i) Assume for now that I really am better at this game than my friend, to the degree that my probability of winning any game that we play is \( \pi = .6 \). Calculate the power of this test, using the \( \alpha = .05 \) significance level. Also be sure to write this probability as \( \Pr(X \geq k) \), where you indicate the appropriate probability distribution of X, and you will in the blank with the appropriate inequality, and you indicate the appropriate value of \( k \).
j) How would the power change if my success probability were larger? Explain why this makes sense intuitively. Then calculate the power when $\pi = .7$, and comment on whether this supports your answer.

k) How would the power change if the significance level were smaller? Explain why this makes sense intuitively. Then calculate the power using $\alpha = .01$ (for an alternative value of $\pi = .6$), and comment on whether this supports your answer.

l) How would the power change if the sample size were larger? Explain why this makes sense intuitively. Then calculate the power using $n = 240$ (with $\alpha = .05$ for an alternative value of $\pi = .6$), and comment on whether this supports your answer.