STAT 301 HW4 due Wed Sept 30

You may work with one partner on this assignment, submitting one report with both names, provided that both students contribute substantially to the work. Word-processed reports are preferred to hand-written ones. Integrate computer output into your report as appropriate.

Cola Discrimination?

A teacher doubted whether his students could distinguish between two different brands of cola soft drink (say, Coke and Pepsi). He presented each of his 24 students with three cups of cola. Two contained the same brand, and the third contained the other brand. Each student was asked to identify the cup containing cola that differed from the other two cups.

Let π represent the probability that a student correctly identifies the "odd" brand. The hypotheses to be tested are H_0: $\pi = 1/3$ vs. H_a: $\pi > 1/3$.

a) Describe (in words) what Type I error means in this situation.

Type I error is to reject H_0 when H_0 is actually true. In this case that means to decide that students do have some ability to discriminate between the colas when in fact they do *not* have any such ability (and so are just guessing).

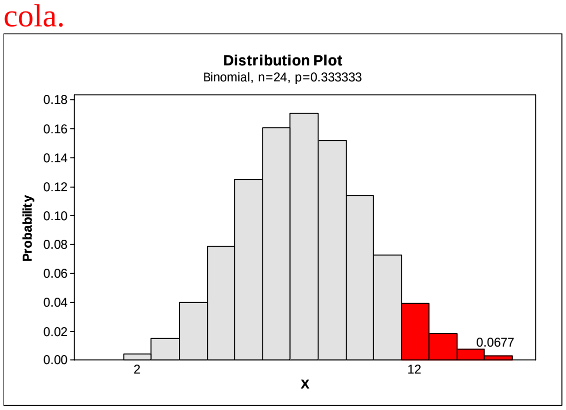
b) Describe (in words) what Type II error means in this situation.

Type II error is to fail to reject H_0 when H_0 is actually false. In this case that means to decide that students have no ability to discriminate between the colas (and so are just guessing) when in fact they *do* have some ability to discriminate.

For the remaining questions, you may use either the Power Simulation applet for an approximate answer or Minitab for an exact answer. (Include screen captures of applet results or Minitab graphs with your answers.)

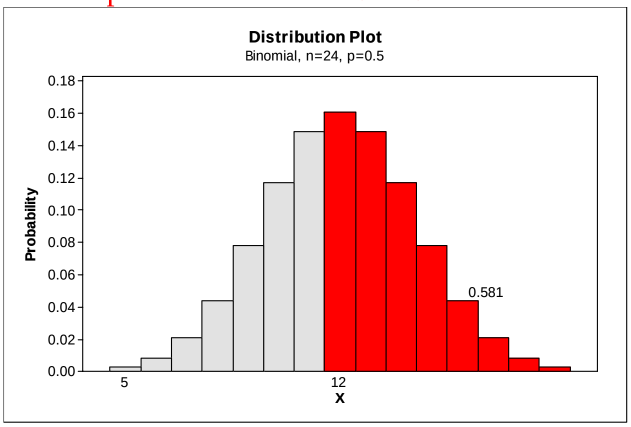
c) Determine the rejection region for this test, using the $\alpha = .10$ significance level.

Using the binomial distribution with $n = 24$ and $\pi = 1/3$, the following Minitab graph reveals that the rejection region is for 12 or more (in the sample of 24 students) to correctly identify the odd cola.



d) Calculate the power of this test, using the $\alpha = .10$ significance level, when the success probability is actually $\pi = .5$.

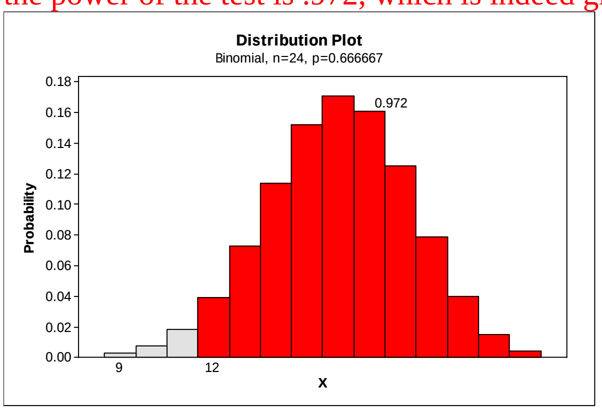
Now using the binomial distribution with $n = 24$ and $\pi = .5$, the following Minitab graph reveals that the power of the test is .581.



e) How would the power change if the success probability were larger? Explain why this makes sense intuitively. Then calculate the power when $\pi = 2/3$, and comment on whether this supports your answer.

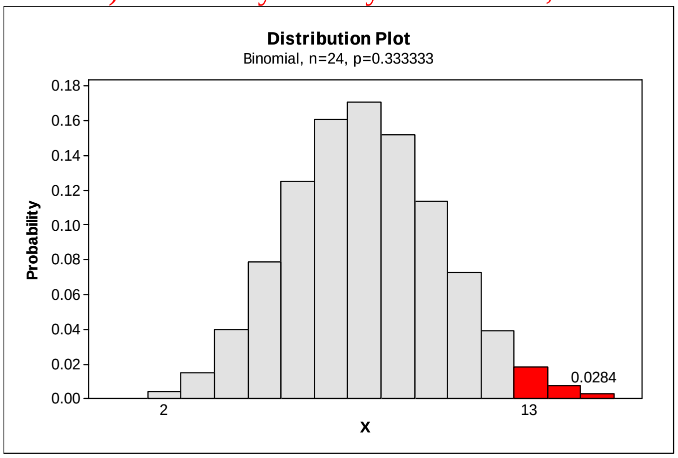
The power should increase. It will be easier (more likely) to detect a difference from pure guessing if the success probability is higher, because the distributions (of number correct) will have less overlap.

Using the binomial distribution with $n = 24$ and $\pi = 2/3$, the following Minitab graph reveals that the power of the test is .972, which is indeed greater than when $\pi = .5$.



f) How would the power change if the significance level were smaller? Explain why this makes sense intuitively. Then calculate the power using $\alpha = .05$ (for an alternative value of $\pi = .5$), and comment on whether this supports your answer.

The power should decrease, because a lower level makes it harder to reject H_0. With a significance level of α = .05, the rejection region is now for 13 or more (in the sample of 24 students) to correctly identify the odd cola, as shown in the following Minitab graph.



Then the power of the test for an alternative value of π = .5 is .419, as shown in the graph below.