STAT 301 HW5 due Mon Oct 5

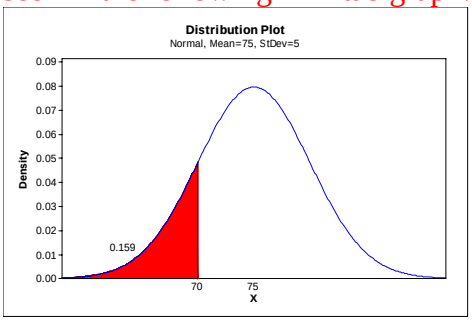
You may work with one partner on this assignment, submitting one report with both names, provided that both students contribute substantially to the work. Word-processed reports are preferred to hand-written ones. Please copy/paste relevant computer output into your report as appropriate.

Drying Paint?

Suppose that the drying time for a certain type of paint under specified test conditions is known to be normally distributed with mean 75 minutes and standard deviation 5 minutes. Suppose that chemists have devised a new additive that is hoped will reduce the mean drying time (without changing the standard deviation). Suppose that a test is conducted to measure the drying time for a test specimen, and suppose that company executives decide that they will be convinced that the additive is effective only if the drying time on this specimen is less than 70 minutes.

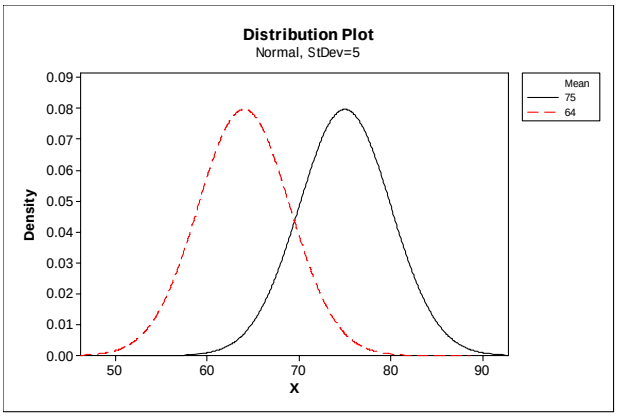
a) If the additive actually has no effect at all on the drying time, what is the probability that the company executives will mistakenly conclude that it is effective? Include a shaded sketch with your calculation, and also report the relevant *z*-score.

We want the probability that a normal distribution with mean $\mu = 75$ and standard deviation $\sigma = 5$ will produce a value below 70. The *z*-score is (70-75)/5 = -1.0. The probability is .159, as seen in the following Minitab graph:



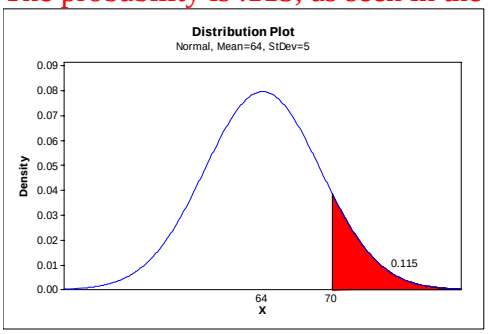
Now suppose that the additive really is effective and that it reduces the mean drying time to 64 minutes, without changing the standard deviation of 5 minutes.

b) Produce a sketch of the two normal curves on the same scale.



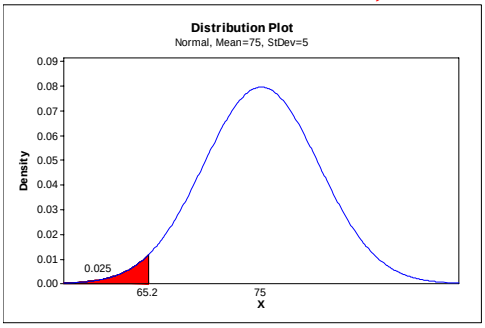
c) What is the probability that this test will fail to convince the executives that the additive is effective, even though it actually is? Again report the relevant *z*-score with your answer.

When the additive really is effective, the mean is $\mu = 64$ and standard deviation is $\sigma = 5$. The executives remain unconvinced if the drying time is 70 or higher. The *z*-score is (70-64)/5 = 1.2. The probability is .115, as seen in the following Minitab graph:



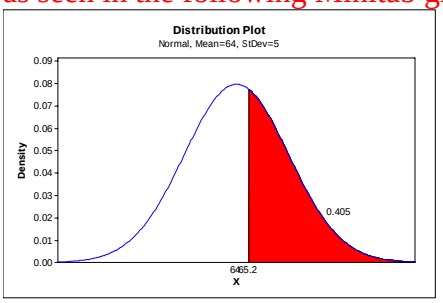
d) If you want alter the cut-off value from 70 in order to reduce the error probability in a) to .025, what cut-off value should you choose?

The error in a) is that executives decide the additive to be effective when it's really not, so the mean is $\mu = 75$ and standard deviation is $\sigma = 5$. The value with probability .025 to its left in this normal curve is 65.2 minutes, as shown in this Minitab graph:



e) Using this new cut-off value from d), what is the probability that that the test will fail to convince the executives that the additive is effective, even though it actually is?

When the additive really is effective, the mean is $\mu = 64$ and standard deviation is $\sigma = 5$. The executives now remain unconvinced if the drying time is 65.2 or higher. This probability is .405, as seen in the following Minitab graph:

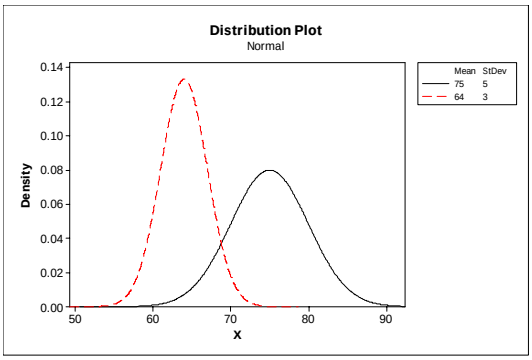


f) How does the probability in e) compare to that in c)? Explain why this makes sense.

This probability has increased substantially (from .115 to .405), because the probability of the other kind of error has been reduced from .159 to .025.

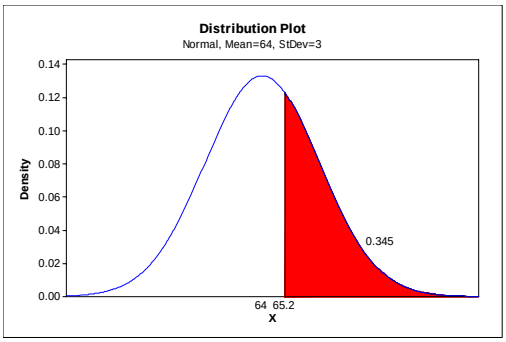
Now suppose that the additive reduces the mean drying time to 64 minutes and also changes the standard deviation to 3 minutes.

g) Produce a sketch of the two normal curves (the original curve assuming that the additive is not effective, and this new one) on the same scale.



h) Still using the cut-off from d), what is the probability that that the test will fail to convince the executives that the additive is effective, even though it actually is?

We now want the probability of taking more than 65.2 minutes, using a normal distribution with mean $\mu = 64$ and standard deviation $\sigma = 3$. This probability is .345, as shown here:



i) How does the probability in h) compare to that in e)? Explain why this makes sense.

The probability has decreased from .405 to .345. It's helpful to reduce the standard deviation of the drying times; this makes it a bit easier to correctly identify when there is improvement.