HW12: Normal distributions
(assigned on Wed Nov 9, due on Mon Nov 14)

You may work with in a group of as many as three students on this assignment, handing in one report with all names, provided that you all contribute to the work.

1. A video display tube for computer graphics terminals has a fine mesh screen behind the viewing surface. The mesh is stretched and welded onto a metal frame during assembly. Too little tension at this stage will cause wrinkles, and too much tension will tear the mesh. Suppose that the tension reading of a randomly selected video display tube has a normal distribution with mean 275 and standard deviation 35 millivolts. Suppose further that the minimum acceptable tension is 230 millivolts and that the maximum acceptable tension is 320 millivolts.

a) Determine and interpret the $z$-score for a tension of 320 millivolts.

b) Express the probability that a randomly selected video display tube has tension within the acceptable range in terms of $\Phi$, the cdf of a standard normal distribution.

c) Determine the probability asked about in part b).

d) Re-answer b), using only positive values for inputs in the function $\Phi$.

e) Now suppose that the manufacturer wants to increase this probability of producing a mesh screen with an acceptable tension to 0.95 by keeping the mean the same and changing the standard deviation. Determine the value of the standard deviation needed to achieve this goal.

f) Continuing with part e), is the new value of the standard deviation larger or smaller than the original standard deviation? Explain why this makes sense intuitively.

2. Suppose that my score on a computer game can be regarded as a continuous random variable following a normal distribution with mean 350 and standard deviation 45.

a) Determine the value such that I score higher than this value 92.5% of the time. Also report the relevant $z$-score for performing this calculation.

b) Determine the probability that my combined score for three independent, randomly selected games exceeds 1000. (As always, show your work.)

c) Suppose that my goal is to exceed 1000 points, and I am offered the choice between
   - playing three independent games, or
   - playing a single game and multiplying my score by 3.
Which choice provides the higher probability of exceeding 1000 points? Justify your answer with appropriate probability calculations.