HW3: Counting techniques  
(assigned on Fri Sept 30, due on Wed Oct 5)

You may work with in a group of as many as three students on this assignment, handing in one report with all names, provided that you all contribute to the work.

1. I once read that a guest on Johnny Carson’s show told him about the birthday problem and said that a group with 50 or more people has a very high probability of finding shared birthdays. Johnny did not believe the guest, so he turned to the audience of about 150 people and said “My birthday is October 23. Is that anyone else’s birthday?” Nobody shared Johnny’s birthday.

a) Explain how Johnny’s question is different from the birthday problem that we solved in class.

b) Under the same simplifying assumptions that we made in class (ignore Feb 29, assume that all days are equally likely to be someone’s birthday), determine the probability that at least one person in a group of 150 people would have birthday of October 23. Is this probability more or less than .5?

c) Now derive a general expression for the probability that at least one person in a group of \( n \) people would share Johnny’s birthday of October 23.

d) Use R to produce a graph of this probability as a function of \( n \), for all values of \( n \) from 1 to 1000. \([Hint: First create the vector \( n \) with the command: \( n=(1:1000) \). Then produce the vector \( prob \) with the appropriate function of \( n \). Then plot the graph with the command: \( plot(n,prob) \).]\\

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e) Determine the smallest value of \( n \) for which this probability exceeds .5. Also report the probability.

f) Comment on how this probability function compares to the one we derived in class for the more classic version of the “birthday problem.”

2. Of the 45 students enrolled in this class, 21 are STAT majors and 24 are not (let’s call these 24 OTHER majors). Suppose that I select 5 of these students at random.

a) Determine the probability that the 5 students selected are all OTHER majors.

b) Determine the probability that OTHER majors outnumber STAT majors among the 5 students selected.

Now suppose that I randomly select 5 students one at a time, without replacement.

c) Determine the probability that the selections alternate between STAT and OTHER majors, in either order, through these 5 selections.
d) Determine the probability that OTHER majors outnumber STAT majors at every stage throughout this random selection process.

3. An Italian engineering company named Ma-Vib employed 12 men and 18 women in early 2011. The company needed to select 15 employees to be laid off, and it turned out that the 15 selected were all women.

a) If the selection of which 15 employees to lay off had been made at random, what is the probability that 15 women would have been selected?

b) Discuss what this probability reveals about the possibility that the company discriminated against women in deciding which employees to lay off.