HW7: Discrete Random Variables
(assigned on Mon Oct 17, due on Thur Oct 20)

You may work with in a group of as many as three students on this assignment, handing in one report with all names, provided that you all contribute to the work.

1. Suppose that 4 men and 4 women take an exam and are then ranked from highest score (rank 1) to lowest score (rank 8). Suppose that no ties occur and that all orderings are equally likely. Let the random variable X = rank for highest-scoring woman.

a) List the possible values of X.

b) Determine the probability for each possible value of X. (As always, show/justify your calculations.)

2. Consider a random variable X with pmf:

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>1/16</td>
<td>1/8</td>
<td>3/16</td>
<td>1/4</td>
<td>3/16</td>
<td>1/8</td>
<td>1/16</td>
</tr>
</tbody>
</table>

a) Determine and graph the cdf of X.

Now consider a random variable Y with cdf:

\[
F(y) = \begin{cases} 
0 & y < 1 \\
1/16 & 1 \leq y < 2 \\
1/4 & 2 \leq y < 3 \\
9/16 & 3 \leq y < 4 \\
1 & y \geq 4 
\end{cases}
\]

b) Determine the pmf of Y.

3. Suppose that I begin the day with 2 umbrellas at home and 1 umbrella in my office. If it’s raining in the morning, then I’ll take one umbrella from my home to the office. If it’s raining in the evening, then I’ll take one umbrella from my office to my home. Suppose that the weather forecast says that there’s a 30% chance of rain tomorrow morning. Additionally, the forecast says that if it’s raining in the morning, then there’s an 80% chance that it will also be raining in the evening. But if it’s not raining in the morning, then there’s only a 10% chance that it will be raining in the evening. Let the random variable Y be the number of umbrellas that I have at my home at the end of the day. Determine the probability mass function (pmf) of Y. [Hints: First calculate probabilities for outcomes such as: rain in the morning and no rain in the evening. As always, show/justify your calculations.]