1. Suppose that you are considering whether to invest in a new business that a friend of yours is starting. After researching the possibilities, you decide that you will gain $50,000 if the business is successful, and you will lose $20,000 if the business is not successful.

   a) Suppose that 30% of all such businesses are successful, so you consider .3 to be the probability that your friend’s business will be successful. Determine the expected value of your net profit.

   Let R denote net profit. \( E(R) = .3(50,000) + .7(-20,000) = 1000 \)

   b) Write a sentence interpreting what this expected value means.

   If you were to make such investments a large number of time, your average profit in the long run would be $1000 per investment.

   c) Now ignore the 30% figure and instead let \( p \) represent the probability that your friend’s business will be successful. Determine the values of \( p \) for which the expected value of your net profit is positive.

   \( E(R) = (p)(50,000) + (1-p)(-20,000) = 70,000p - 20,000 \). Setting this > 0 and solving for \( p \) gives: \( p > 2/7 \approx .286 \).

2. Suppose that among the pre-owned cars available in a large dealership, 80% have air conditioning, 70% have a CD player, and 65% have both.

   a) Determine the proportion of these cars that have neither air conditioning nor a CD player. (As always, show your method of solution.)

   The probability table starts with the given information:

<table>
<thead>
<tr>
<th></th>
<th>AC</th>
<th>No AC</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>.65</td>
<td>.05</td>
<td>.7</td>
</tr>
<tr>
<td>No CD</td>
<td>.15</td>
<td>.15</td>
<td>.3</td>
</tr>
<tr>
<td>Total</td>
<td>.8</td>
<td>.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

   and then fills in as:

<table>
<thead>
<tr>
<th></th>
<th>AC</th>
<th>No AC</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>.65</td>
<td>.15</td>
<td>.7</td>
</tr>
<tr>
<td>No CD</td>
<td>.15</td>
<td>.15</td>
<td>.3</td>
</tr>
<tr>
<td>Total</td>
<td>.8</td>
<td>.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

   The probability of having neither AC nor a CD is .15.

   b) Determine the conditional probability that a randomly selected car has a CD player, given that it has air conditioning.

   \( P(CD | AC) = P(CD and AC) / P(AC) = .65 / .8 = .8125 \)
c) Are the events \{randomly selected car has air conditioning\} and \{randomly selected car has a CD player\} independent? Justify your answer.

No, because \(P(CD | AC) = 0.8125\) but \(P(CD) = 0.7\), so knowledge of one event changes the probability of the other.

d) Are the events \{randomly selected car has air conditioning\} and \{randomly selected car has a CD player\} disjoint? Explain briefly.

No, because 65% of the cars have both AC and CD.

3. A teacher conducted a study to see whether students could distinguish between the tastes of two brands of cola. Each of 30 students was presented with three cups: Two cups contained the same brand of cola, and the third cup contained a different brand of cola. Each student was asked to identify which cup contained the different brand. Let the random variable \(X\) represent the number of students who correctly identify the different cup.

a) If students are really not able to distinguish, and so they randomly choose among the three cups, what would be the probability of correctly identifying the different one?

\(1/3\)

b) It turned out that 16 of 30 students correctly identified the different cup. Fill in the following blanks to specify how to calculate the p-value for this study:

\[
p\text{-value} = P(X \geq 16), \text{ where } X \text{ has a binomial distribution}
\]

with parameter values \(n = 30\) and \(p = 1/3\).

c) The p-value turns out to equal .019. Finish the following sentence to interpret this p-value:

The probability is .019 that

16 or more of the 30 students would have correctly identified the different cup

assuming that

the students really have no ability to distinguish cola tastes.

d) Based on this p-value, would you conclude that the sample data provide fairly strong evidence that students really can identify the different cola more often than by random chance? Also explain the reasoning process behind your answer.

Yes. This p-value of .019 is quite small, so the observed result (16 of 30 correct) would happen quite rarely (less than 2% of the time) if in fact students had no ability to distinguish cola tastes.
So, because the observed result would be surprising if students had no such ability, we have fairly strong evidence that students do have some ability to distinguish cola tastes better than random chance.

4. Suppose that when the interest rate goes up in a particular month, stock prices go down with probability .8 and stock prices go up with probability .2. Also suppose that when the interest rate goes down in a given month, stock prices go down with probability .3 and stock prices go up with probability .7. Finally, suppose that the probability is .4 that the interest rate will go up next month and .6 that the interest rate will go down next month.

Use the following symbols for these events:

\[ \text{SU} = \{ \text{stock price goes up} \} \quad \text{SD} = \{ \text{stock price goes down} \} \]
\[ \text{IU} = \{ \text{interest rate goes up} \} \quad \text{ID} = \{ \text{interest rate goes down} \} \]

a) Express the following three probabilities from the paragraph above in terms of these event symbols:

\[ .8 = P(\text{SD} \mid \text{IU}) \]
\[ .3 = P(\text{SD} \mid \text{ID}) \]
\[ .4 = P(\text{IU}) \]

b) Determine the probability that the interest rate goes down and stock prices go up next month. Justify your answer with a probability tree or a probability table or a probability rule.

By the multiplication rule, \( P(\text{ID} \text{ and } \text{SU}) = P(\text{ID}) \times P(\text{SU} \mid \text{ID}) = (.6)(.7) = .42. \)

c) Determine the probability that stock prices go up next month.

By the law of total probability, \( P(\text{SU}) = P(\text{ID}) \times P(\text{SU} \mid \text{ID}) + P(\text{IU}) \times P(\text{SU} \mid \text{IU}) = (.6)(.7) + (.4)(.2) = .50. \)

5. Suppose that Jose has applied for a job with three different companies. He thinks that he has a 40% chance of getting a job offer from company A, a 60% chance of getting a job offer from company B, and a 90% of getting a job offer from company C. Furthermore, he believes that whether or not he receives a job offer from any company is independent of whether or not he receives a job offer from any other company.

a) What is the probability that Jose receives a job offer from all three companies?

\[ P(\text{A and B and C}) = P(A) \times P(B) \times P(C) \text{ because of independence, which } = (.4)(.6)(.9) = .216 \]

b) What is the probability that Jose receives a job offer from at least one of these three companies?

\[ P(\text{A or B or C}) = 1 – P(\text{not A and not B and not C}) \]
\[= 1 - (1-.4)(1-.6)(1-.9) = 1 - (.6)(.4)(.1) = 1 - .024 = .976\]

6. Scores on the verbal ability portion of the Graduate Record Examination (GRE) follow a normal distribution with mean 500 and standard deviation 115.

a) Between what two values do the middle 95% of scores fall?

The empirical rule establishes that about 95% of the scores fall within two standard deviations of the mean, so within \(\pm 2(115) = 230\) of 500. So, 95% of scores fall between 500 – 230 = 270 and 500 + 230 = 730.

A more exact answer results from using 1.96 rather than 2: \(500 - 1.96(115) = 274.6\) and \(500 + 1.96(115) = 725.4\).

b) If your score is 750, then you did better than what percentage of exam takers?

The \(z\)-score is \((750-500)/115 \approx 2.17\). Looking this up in the standard normal table reveals that you did better than .9850 (or 98.50%) of GRE takers.