Comparing Two Proportions

Now we learn how to conduct statistical inference when comparing results between two groups, which greatly expands the kinds of questions that we can investigate.

Example 19-1: AZT for HIV (cont.)  
One of the first studies aimed at preventing maternal transmission of AIDS to infants gave the drug AZT to pregnant, HIV-infected women in 1993. Roughly half of the women were randomly assigned to receive the drug AZT, and the others received a placebo. The HIV-infection status came to be known for 363 babies, 180 from the AZT group and 183 from the placebo group. Of the 180 babies whose mothers had received AZT, 13 were HIV-infected, compared to 40 of the 183 babies in the placebo group.

a) Is this an observational study or a randomized experiment? Explain how you know.

b) Identify the explanatory and response variables in this study. What types of variables are these?

c) Organize these data into a two-way table, with the explanatory variable in columns and the response in rows.

d) What proportion of the women given AZT had HIV+ babies? What about for the women in the control group? Use appropriate symbols. Does this difference appear to be large?

e) State the null and alternative hypotheses for the AZT study, in symbols and in words.

We will learn a procedure for testing whether two sample proportions differ significantly:
- The null hypothesis is $H_0: \pi_1 = \pi_2$
- The alternative hypothesis is $H_a: \pi_1 > \pi_2$ or $H_a: \pi_1 < \pi_2$ or $H_a: \pi_1 \neq \pi_2$. 
The test statistic is 

\[ z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c (1 - \hat{p}_c) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}. \]

The \( p \)-value is the area under the standard normal curve more extreme than the test statistic, in the direction indicated by the alternative hypothesis.

The test decision is to reject \( H_0 \) whenever \( p \)-value \( < \alpha \) (significance level).

The technical conditions required for this procedure to be valid are:
- independent random samples from the populations, or randomly assigned groups
- at least 10 “successes” and at least 10 “failures” in each group

e) What proportion of the mothers (for the two groups combined) had HIV+ babies?

f) Calculate the test statistic and \( p \)-value.

g) Use Minitab (Stat> Basic Statistics> 2 Proportions...) or the Theory-Based Inference applet to confirm these calculations.

h) What conclusion would you draw from this test? Explain how this conclusion follows from your test result.

i) Can you legitimately draw a cause-and-effect conclusion between AZT and the baby’s HIV status?

Note that the hypothesis test assesses how much evidence the data provide that AZT and placebo produce different results, but it does not say anything about how different the results are. To address this question, we can estimate the difference \( \pi_1 - \pi_2 \) with a confidence interval:

\[ \left( \hat{p}_1 - \hat{p}_2 \right) \pm z \* \sqrt{\frac{\hat{p}_1 (1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 (1 - \hat{p}_2)}{n_2}}. \]
j) Produce a 95% confidence interval for estimating the difference in HIV+ rates between the
two groups $\pi_{\text{plac}} - \pi_{\text{AZT}}$.

k) Interpret this interval. Pay particular attention to whether it includes zero or is entirely
positive or entirely negative.

l) What would be different if you instead found a 95% confidence interval for $\pi_{\text{AZT}} - \pi_{\text{plac}}$?
Explain.

Example 19-2: Pet CPR
A recent survey of pet owners, found that 53% of cat owners and 63% of dog owners said that
they would perform CPR on their pets in the event of a medical emergency.

a) Are these numbers parameters or statistics? What symbols would we use to represent these
numbers?

b) State the appropriate null and alternative hypotheses for testing whether the difference
between 53% and 63% is statistically significant in this context. Use appropriate symbols, and
also explain what the symbols represent in this context.

c) What additional information would you need in order to conduct a test of these hypotheses?
Suppose for now that the sample sizes had been 100 in each group.

d) Calculate the $z$-test statistic by hand. Also determine the p-value. Would you reject the null hypothesis at the .05 significance level?

e) Determine by hand a 95% confidence interval for comparing the two proportions. Also write a sentence or two interpreting this interval.

f) Are the test decision and confidence interval consistent with each other? Explain how you can tell.

g) Now suppose that the sample sizes had been 500 in each group. Calculate the $z$-test statistic, p-value, and confidence interval. (Feel free to use software.) Summarize your conclusions.

h) Summarize what this example reveals about the role of sample size in determining whether a difference between two sample proportions is statistically significant.