Stat 321 - Day 25
Other Continuous Distributions
(Gamma, Exponential, Weibull)

Example: Battery Lifetimes
The Minitab worksheet `lifelengths.mtw` contains data on lifetimes of batteries.

(a) Examine a visual display (histogram and/or dotplot) of these data. Comment on the shape. Do you think that a normal distribution would provide a good model for battery lifetimes? Explain.

While the normal distribution is the most widely studied and applied continuous probability distribution, not all data follow a bell-shaped curve.

Gamma Distribution
Properties: Wide variety of skewed shapes
Often useful to model lifetimes Domain: $x \geq 0$
Calculate probabilities using Minitab with parameters: α, β=shape parameter, or:
standardizing (divide x by β) and using Table A.4
$E(X)=\alpha\beta$, $V(X) = \alpha\beta^2$

Example: Maintenance Check Times
Suppose that the length of time to conduct a period maintenance check (from previous experience) on a dictating machine follows a gamma-type distribution with $\alpha=3$ and $\beta=2$ (min).
(b) Define an appropriate random variable:

X =

(c) Determine the mean length of maintenance times.

(d) Suppose that a new repairman requires 20 minutes to check a machine. Determine the probability of a time at least this long based on prior experience. Fill in the following:

$P(X \geq$ ______ $) = 1 - F(x/\beta=$ ______ with $\alpha=3) = 1-$ ______ from Table A.4 = ______ .

Special Case: Exponential Distribution
A Gamma Distribution with $\alpha=1$ and $\beta=1/\lambda$
Often used to model interarrival times
Calculate probabilities using Minitab (giving mean) or:
$f(x) = \lambda e^{-\lambda x}$ $x \geq 0$; 0 otherwise. Parameter: λ
$F(x) = 1-e^{-\lambda x}$ $x \geq 0$; $E(X)=1/\lambda$ $V(X)=1/\lambda^2$

Example: Battery Lifetimes (cont.)

(e) Open the Excel file `exponential.xls`. The pink graph represents the histogram of lifetimes. The blue curve represents the exponential pdf. Change the value of λ specified in F2 and hit return. Did you increase or decrease λ? How did the blue curve change?

(f) Try several values of λ until you believe the blue curve models the pink graph well. What value of λ did you decide on?

(g) Use your value of λ to calculate E(X) according to the formula above. How does this value of E(X) compare to the sample mean of the 50 observations?

(h) Use the cdf given above and your value of λ, to determine the probability a battery would last more than 6 (hundred hours).

(i) Open the Excel file `weibull.xls`. The data in column B are the length of service during which a certain type of thermistor produces resistances within its specifications (in thousand hours). Column G controls $g(x)$, an exponential pdf. Change the value of λ to see if you can provide a reasonable match for the data. Do you think it's reasonable to model these data with the exponential distribution?

Weibull distribution
Properties: Very flexible; often used to model lifetimes.
Calculate probabilities using Minitab or:

$F(x) = 1-e^{-(x/\beta)^\alpha}$ $\quad x \geq 0$ $\quad$ Parameters: α (shape), β (scale)

Special case: Exponential = Weibull(α=1, β=1/λ)

Example: Length of Service

(j) In `weibull.xls` try different values of α and β to see if you can find a Weibull distribution that provides a more reasonable match to the data. (You might want to keep in mind that β is considered a scale parameter and α a shape parameter.) What were your final choices for α and β?

(k) What do you notice about the pdf when α=1? Does it look familiar? Explain.

(l) Use your values of α and β and the above cdf formula to calculate the probability that a service is less than 1 (thousand hours).