Fall, 2004 *Tuesday, September 28*

Stat 321 - Day 6
Conditional Probability and Independence

Recall that the conditional probability of an event B given that an event A has occurred is give by: $P(B|A) = P(A \cap B)/P(A)$.

Example: Top 100 Films (cont.)
Suppose that one of the 100 films is chosen at random. Recall that the following table reports the number of films in the AFI Top 100 list seen by Allan and by Beth:

	Beth yes	Beth no	Total
Allan yes	42	6	
Allan no	17	35	
Total			100

(a) Determine the (unconditional) probability that Allan has seen the film.

(b) Determine the conditional probability that Allan has seen the film given that Beth has.

(c) Does the knowledge that Beth has seen the film make it more or less likely that Allan has seen it, or does that probability not change given the knowledge that Beth has seen the film?

Now consider hypothetical data representing the number of these films seen by Chuck and by Donna:

	Donna yes	Donna no	Total
Chuck yes	15	10	
Chuck no	45	30	
Total			100

(d) Compare Donna's (unconditional) probability of having seen the film with the conditional probability that she has seen it given that Chuck has. Does the knowledge that Chuck has seen the film change the probability that Donna has seen it?

$P(D) =$ $P(D|C) =$

Two events A and B are said to be *independent* if $P(A|B)=P(A)$; otherwise they are *dependent*.

(e) Are the events {Allan has seen the film} and {Beth has seen it} independent? How about {Chuck has seen the film} and {Donna has seen it}? Explain.

(f) Algebraically derive an equivalent expression for independence that involves P(A∩B), P(A), and P(B).

Now suppose you are told that Ellen has seen 80% of the films that Donna has seen.

(g) Express this value of .8 as a conditional probability involving the events E = {Ellen has seen it} and D = {Donna has seen it}.

(h) Can you use the information given about Donna and Ellen to determine the proportion of films that have been seen by both Donna and Ellen? If so, please do. [*Hint*: Solve for P(D∩E) from the expression for P(E|D).]

The *multiplication rule*, which follows immediately from the definition of conditional probability, asserts that: P(A∩B) = P(A) P(B|A). This can equivalently be written as: P(A∩B) = P(B) P(A|B). When the events are *independent*, this becomes P(A∩B) = P(A) P(B).

(i) Explain how the multiplication rule for independent events follows from the more general multiplication rule.

Example: Graduate School Admissions

Suppose that you apply to two graduate schools A and B, and that you believe your probability of acceptance by A to be .7, your probability of acceptance by B to be .6, and your probability of acceptance by both to be .5.

(j) Are the events {acceptance by A} and {acceptance by B} independent? Explain. [*Hint*: Use the alternative definition that you derived in (f).]

(k) Determine the conditional probability of acceptance by B given acceptance by A. How does it compare to the (unconditional) probability of acceptance by B?

(l) What is the probability that you are accepted by at least one of the two schools?

Now suppose that the events {acceptance by A} and {acceptance by B} are independent, with probability of acceptance by A equal to .7 and probability of acceptance by B equal to .6.

(m) Determine the probability of acceptance by both schools. Then determine the probability of acceptance by at least one school. Also indicate appropriate symbols and set operations to describe these events.

The multiplication rule for a series of *independent* events A_1, A_2, …, A_k asserts that $P(A_1 \cap A_2 \cap \ldots \cap A_k) = P(A_1)P(A_2)\ldots P(A_k)$.

Suppose that you also apply to graduate schools C and D, that you consider all acceptances to be independent of each other, and that you believe the probabilities of acceptance to be .8 and .5, respectively.

(n) Determine the probability of acceptance by all four schools.

(o) Determine the probability of acceptance by at least one of the four schools. [*Hint*: First find the probability of the complement of this event.]

Example: Foul Shooting

Suppose that a basketball player makes 80% of his foul shots, independently from shot to shot. Suppose further that he is to take one shot, and then if he makes that shot he gets to take another shot. If he misses the first shot, he does not get to take another shot.

(p) Determine the probability that he makes both shots.

(q) Determine the probability that he makes exactly one shot.

(r) Repeat (p) and (q) for a player with a success probability of p.

Now suppose that the 80% shooter faces two of these situations in a game.

(s) Determine the probability that he makes exactly two foul shots.

Example: Rolling Dice

One of the famous historical questions that led to the development of probability theory is:

(t) Which is more likely- that a 6 will be rolled at least once in four independent rolls of a fair die, or that a double 6 will be rolled at least once in 24 independent rolls of a pair of fair dice?