Stat 321 - Day 7
Law of Total Probability

Example: Governor Election
According to exit polls conducted by CNN during the 2003 California election, Governor Schwarzenegger won 52% of the white votes, 17% of the black votes, 31% of the Hispanic votes, and 37% of the votes from other races. Suppose that one of the voters is selected at random.

(a) Is it correct to find the (unconditional) percentage who voted for Schwarzenegger by taking the average of these four percentages? Explain why or why not. If not, indicate under what conditions that would be correct.

(b) Does knowing that 70% of those interviewed were white change your thinking about (a)?

(c) Let A denote the event that a voter voted for Schwarzenegger. Let W denote the event that a voter was white, B that a voter was black, H that a voter was Hispanic, and O that a voter was another race. Translate the above percentages into probability statements using the symbols given in parentheses.

.52 = .17 = .31 = .37 = .70 =

The CNN exit poll results further revealed that 70% of those interviewed were white, 6% were black, 18% Hispanic, and 6% other races.

(d) Record these as (unconditional) probabilities in the bottom row of the *probability table* below.

	White (W)	Black (B)	Hispanic (H)	Others (O)	Total
Arnold (A)					
Others (A')					
Total					1.00

(e) Use the multiplication rule to find $P(W \cap A)$. Also express this event in words. Then record this probability in the upper left cell of the probability table.

(f) Similarly calculate the other intersection probabilities that comprise the first row of the probability table.

(g) Sum those four probabilities in the first row of the table to determine the (unconditional) probability of voting for Schwarzenegger.

You have derived the *Law of Total Probability (LTP)*, which enables you to calculate an unconditional probability when you knows conditional probabilities. It says that if the events A_1, $A_2, \ldots, A_k$ are mutually exclusive and exhaustive (i.e., they form the entire sample space with their union), then for any event *B*: $P(B) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$. In other words, the probability of B is a weighted average of the conditional probabilities given a certain group designation, with weights equal to the group probabilities.

(h) Create a probability tree to represent the calculation of the (unconditional) probability of voting for Schwarzenegger.

Example: Multiple Choice Exam

Suppose that a student has a 50% chance of knowing (with certainty) the answer to a question on a multiple choice exam. Suppose further that when he does not know the answer, he guesses at random among the 4 choices.

(i) First define the relevant events in this problem.

(j) Use the LTP to determine the LTP to determine what proportion of questions the student would answer correctly in the long run (i.e., the probability that the student answers a randomly selected question correctly). [*Hint*: You may use a probability tree, probability table, or the rule itself.]

(k) Repeat (j) if there are *k* choices (not necessarily 4) on each multiple choice question.

(l) Is this an increasing or decreasing function of *k*? Explain why that makes sense.