STAT 325 – Handout 16
Expectations for Continuous Random Variables (5.2)

- The **expected value** of a continuous random variale X with pdf $f(x)$ is calculated by: $\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$.
  - This corresponds to the center of mass, or balance point, of the pdf.
  - This is interpreted in the same way as for a discrete random variable: long-run average value of X if the random process is repeated a very large number of times.
- Expected value of some function g(X) is: $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$
- Variance and standard deviation are defined as before: $\sigma^2 = Var(X) = E[(X-\mu)^2]$ and SD(X) = sqrt[Var(X)]
  - Short-cut formula still holds: $Var(X) = E(X^2) – [E(X)]^2$
- Previous rules for expectations and variances still hold:
  - $E(aX + b) = a E(X) + b$ for any constants a and b, any random variable X
  - $E(X + Y) = E(X) + E(Y)$ for any random variables X and Y
  - $E(a_1X_1 + a_2X_2 + \ldots + a_kX_k) = a_1E(X_1) + a_2E(X_2) + \ldots + a_kE(X_k)$
  - $Var(aX + b) = a^2Var(X)$
  - If X and Y are independent, then $Var(X + Y) = Var(X) + Var(Y)$.

Example 16-1: Random lunches (cont.)
Reconsider the four probability distributions for the time (in hours after noon) at which a businessperson leaves for lunch.

a) Based on the graphs of the pdf's, with which distribution would you predict the expected time of leaving to be largest? Explain.

b) Based on the graphs of the pdf's, with which distribution would you predict the variance to be largest? Explain.

c) Based on the graphs of the pdf's, with which distribution would you predict the variance to be smallest? Explain.

d) Explain why it makes sense that $E(X_1) = E(X_3) = E(X_4) = .5$.

e) Calculate $E(X_2)$. Is this larger, smaller, or the same as the other expected values?

Some of the variances turn out to be: $\mathrm{Var}(X_1) = 1/12 \approx .0833$, $\mathrm{Var}(X_2) = 5/18 \approx .2778$, and $\mathrm{Var}(X_3) = 1/24 \approx .0417$.

f) Calculate $\mathrm{Var}(X_4)$.

g) Determine the probability that X_4 falls within one standard deviation of its mean.

Example 16-2:
a) Determine the expected value of a uniform random variable on the interval (a, b).

b) Explain what this expected value represents geometrically.

c) Determine the variance of a uniform random variable on the interval (a, b).

d) On what aspect of the interval (a, b) does the variance depend? Explain why this makes sense.

Example 16-3:
Suppose that a random variable X has pdf $f(x) = e^{-x}$, $x > 0$ and $f(x) = 0$ otherwise.

a) Sketch this function, and confirm that it is a legitimate pdf.

b) Determine E(X). [*Hint*: Use integration by parts and L'Hopital's rule.]

c) Determine $\Pr[X \leq E(X)]$. Does this equal .5? Is it close? Does this make sense? Explain.

d) Determine the value *m* such that $\Pr(X \leq m) = .5$. [Note: *m* is called the *median* of this random variable.]

e) What can you say about $\Pr(X \geq m)$? Justify every step of your answer, without performing any calculus.

f) Determine $E(X^2)$ and Var(X).