STAT 325 – Handout 7
Independence (1.6)

One of the most important and useful concepts in probability is *independence*. The basic idea is that events are independent if learning about the occurrence (or non-occurrence) of one event does not change the probability of the other event. The following three conditions are all equivalent definitions:

- Two events E and F are **independent** if:
  - Pr(E|F) = Pr(E)
  - Pr(F|E) = Pr(F)
  - Pr(E ∩ F) = Pr(E) × Pr(F)
    - This last equality is called the **multiplication rule** for *independent* events.
- Two events that are not independent are said to be **dependent**.

Sometimes we check whether given probabilities imply independence between events, and sometimes we assume that events are independent so we can calculate the probability of their intersection by multiplying their individual probabilities.

Example 7-1: Top 100 films (cont.)
Recall again the following table:

	Beth yes	Beth no	Total
Allan yes	42	6	48
Allan no	17	35	52
Total	59	41	100

Suppose again that a film is chosen at random. Are the events {Allan has seen the film} and {Beth has seen the film} independent? Justify your answer.

Example 7-2: Rolling Dice (cont.)
Roll two fair dice. Which pairs of the following events are independent and which are not?

A = {sum is 7} B = {sum is 11} C = {1^{st} number is 6}

- *General multiplication rule* for independent events: If E_1, E_2, …, E_k are independent events, then $\Pr(E_1 \cap E_2 \cap \ldots \cap E_k) = \Pr(E_1) \times \Pr(E_2) \times \Pr(E_3) \times \ldots \times \Pr(E_k)$.

Example 7-3: Graduate School Applications (cont.)
Suppose that you have applied to two graduate schools E and F, and you assess your probability of being accepted by E as .6 and by F as .7. You also consider these to be independent events.

a) Determine the probability that you are accepted by both schools.

b) Determine the probability that you are accepted by at least one school.

Now suppose that you have also applied to graduate schools G and H, with acceptance probabilities of .4 and .9, respectively. You consider all events to be independent of each other.

c) Determine the probability that you are accepted by all four schools.

d) Determine the probability that you are accepted by at least one of the four schools. [*Hint*: First find the probability of the complement.]

Example 7-4: Daily Lottery
Suppose that every day a person plays a lottery game that has a 1/1000 probability of winning. Determine the probability that the person wins at least once if he plays every day for:

a) a 7-day week

b) a 31-day month

c) a 365-day year

d) Determine and graph the probability of winning at least once, as a function of the number of days *n*.

e) For how many days must the person play to have probability > .5 of winning at least once? [Determine the answer both analytically and by examining the graph.]

f) For how many days must the person play to have probability > .9 of winning at least once?

Example 7-5: Unusual Dice

Consider four six-sided dice with unusual numbers on their sides, as follows:

- Die A: 4, 4, 4, 4, 0, 0
- Die B: 3, 3, 3, 3, 3, 3
- Die C: 6, 6, 2, 2, 2, 2
- Die D: 5, 5, 5, 1, 1, 1

Assume you and I choose two dice to roll independently, and the one producing the larger number wins. Being considerate, I'll ask you to choose your die first.

For whichever pair of dice that you and I choose, determine the probability that the number you roll will be larger than the number that I roll.

Example 7-6: World Series

Suppose that two sports teams (A and B) play a series of games, with the first team to win two games being declared the winner. (This is a "best-of-three" series.) Assume that team A has probability *p* (where $p > .5$) of beating team B in any one game and that the games are independent.

a) The outcomes in the sample space are: S = $\{A_1A_2, A_1B_2A_3, A_1B_2B_3, B_1B_2, B_1A_2B_3, B_1A_2A_3\}$. Are these equally likely? If $p = .5$, are they equally likely? Explain.

b) Which of these outcomes comprise the event that team A wins the series?

c) Determine the probability that team A wins the series, as a function of *p*.

d) Graph this function.

e) Evaluate this probability for $p = .6$, $p = .7$, and $p = .8$.

f) Would you expect team A's probability of winning the series to be larger, smaller, or the same in a best-of-five series, in which the first team to win three games is the winner, as opposed to a best-of-three series? Explain your thinking.

g) Determine the probability that team A wins a best-of-five series.

h) If you are a fan of team B, would you prefer a 1-game series, a 3-game series, or a 5-game series? Explain.

Example 7-7: Solitaire (cont.)
Suppose that I have a .2 probability of winning a game of Solitaire, independently from game to game. I like to play until I win for the first time. Determine the probability that I'll need to play more than 5 games (which is 1/.2) to achieve this goal. [*Hint*: Think about whether it might be easier to determine the probability of the complement first.]

Example 7-8: Foul Shooting Contest (cont.)
Suppose that Jose has a .4 probability of making a foul shot, and Karla has a .8 probability. Jose shoots first, and then they alternate until one of them makes a shot. Whoever makes a shot first wins the contest.

a) Make a guess for the probability that Jose wins this contest.

b) Describe what assuming independence means in this context.

c) Determine the probability that Jose wins this contest on his first shot.

d) Determine the probability that Jose wins this contest on his second shot. [*Hint*: Think about what has to happen on earlier shots for both players.]

e) Determine the probability that Jose wins this contest on his third shot.

f) Use an infinite sum to determine the probability that Jose wins this contest.