STAT 325 – Handout 8
Discrete Random Variables (2.1)

- A **random variable** is one whose value is determined by some chance mechanism. It can be thought of as a function that takes an outcome in a sample space as input and then gives a real number as output.
  - With a **discrete** random variable, the set of possible values is either finite or countably infinite.
  - With a **continuous** random variable, the probability is spread continuously over intervals of real numbers.
- Random variables are often denoted with capital letters, toward the end of the alphabet.
  - Lower case letters denote possible *values* of random variables.

Example 8-1: Matching Babies (cont.)
Recall from Example 7 (handout 3) the random process that returns four babies to their mothers at random. The sample space of possible outcomes can be represented as:

1234	1243	1324	1342	1423	1432
2134	2143	2314	2341	2413	2431
3124	3142	3214	3241	3412	3421
4123	4132	4213	4231	4312	4321

Let the random variable X = number of mothers who get the correct baby.

a) For each of the 24 outcomes in the sample space above, determine the numerical value that the random variable X assigns to that outcome.

b) List all possible values of the random variable X. Then report the probability for each of those possible values.

- The **probability mass function** (pmf) of a discrete random variable X is a function that assigns a probability to each possible value of X.
  - The pmf of X is denoted by $p(x)$, where $p(x) = \Pr(X = x)$.
    - $p(x) \geq 0$ for all x
    - $\sum_x p(x) = 1$

c) Write out the pmf of the random variable X in this case.

d) For the following values *x*, report $\Pr(X \le x)$. [Note that this is not asking for $\Pr(X = x)$.]

X	0	1	2	4
$\Pr(X \le x)$				

e) For the following real numbers *x*, report $\Pr(X \le x)$.

X	-1	1.5	π (3.14159 ...)	56.2897
$\Pr(X \le x)$				

- The **cumulative distribution function** (cdf) of a (discrete or continuous) random variable X is a function that takes any real number as input and outputs the probability that the random variable is *less than or equal to* the input value.
  - The cdf is typically denoted by a capital letter and is defined for all real numbers *x* by: $F(x) = \Pr(X \le x)$.
  - The cdf is a non-decreasing function.
  - The cdf approaches 0 as the input approaches negative infinity.
  - The cdf approaches 1 as the input approaches positive infinity.

f) Write out the cdf for this random variable X.

g) Create a graph of this cdf. How would you describe such a graph/function?

- The cdf of a discrete random variable is a *step function*.
  - The steps occur at the possible values of the random variable.
  - The height of a particular step corresponds to the probability of that value.

Example 8-2: Rolling Dice (cont.)

a) Consider a random variable X with pmf given by: $p(x) = \begin{cases} 5/12 & x = -1 \\ 1/6 & x = 0 \\ 5/12 & x = 1 \\ 0 & otherwise \end{cases}$

Determine and graph the cdf of X.

b) Consider a random variable Y with cdf given by: $F(y) = \begin{cases} 0 & y < 0 \\ 1/6 & 0 \le y < 1 \\ 4/9 & 1 \le y < 2 \\ 2/3 & 2 \le y < 3 \\ 5/6 & 3 \le y < 4 \\ 17/18 & 4 \le y < 5 \\ 1 & y \ge 5 \end{cases}$

Determine and graph the pmf of Y.

Example 8-3: Solitaire

Recall the scenario from Example 1-4 that every night I play Solitaire on my computer until I win for the first time. Let's suppose that my probability of winning any one game is 1/9 and that the results of the games are independent.

a) Write down five of the outcomes in the (infinite) sample space.

Let the random variable Z = number of games that I play in order to achieve my first win.

b) What are the possible values of Z?

c) For each of the outcomes that you wrote in a), report the value that this random variable assigns to that outcome.

d) Determine the probability for each of the outcomes that you wrote in a).

e) Determine and graph the pmf of Z.

f) Verify that the pdf sums to 1. Also indicate the calculus tool needed to do this.

g) Determine and graph the cdf of Z.