STAT 325 Introduction to Probability Models Spring 2010

HW10 due Wed Apr 28

Topics: Binomial distribution

1. Suppose that 8 people get together at lunchtime to play volleyball. First they have to decide how to divide themselves into 2 teams of 4 players each. Plan A is for each person to flip a fair coin. If the flips result in 4 heads and 4 tails, then that determines the teams. If the flips do not result in 4 heads and 4 tails, then they all flip their coins again and repeat until they get a 4-4 result.

a) Determine the probability that Plan A successfully forms 2 teams of 4 players after just 1 set of coin flips.

Let X be the number of the 8 coin flips that land heads, so X ~ Binomial(n = 8, p = .5).

$$\Pr(X = 4) = \binom{8}{4}(.5)^4(1-.5)^4 \approx .2734$$

Plan B is for the first 7 people who show up to flip a fair coin. If it turns out that 4 get heads and 3 get tails, or 3 heads and 4 tails, then that determines the teams, with the last person to arrive joining the team with 3.

b) Determine the probability that Plan B successfully forms 2 teams of 4 players after just 1 set of coin flips.

Let Y be the number of the 7 coin flips that land heads, so Y ~ Binomial(n = 7, p = .5).

$$\Pr(Y = 3) + \Pr(Y = 4) = \binom{7}{3}(.5)^3(1-.5)^4 + \binom{7}{4}(.5)^4(1-.5)^3 \approx .2734 + .2734 = .5468$$

c) Is plan B better than plan A? Explain.

Yes. Plan B is twice as likely to successfully form teams with just one set of flips.

2. I am in a “fantasy golf” league with 4 other players. Each week we select a team of professional golfers and score points based on how our golfers perform in that week’s tournament. At the end of the week, one of the 5 members of my league is that week’s winner. (We’ll ignore the possibility of a tie.) In the first 12 weeks of this season, I was the winner in 7 weeks.

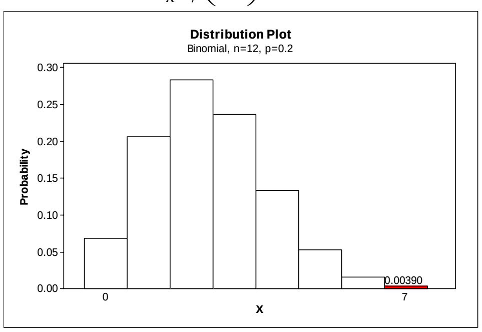
Let's assume for now that all 5 members of my league have the same probability of winning each week, independently from week to week. Let the random variable X be the number of times that I win in the first 12 weeks of the season.

a) Under the equal likeliness assumption, what kind of random variable is X? (Specify its parameter values as well as its name.)

X ~ Binomial(n = 12, p = 1/5 = .2)

b) Write an expression for the probability that I would win 7 or more times in the first 12 weeks of the season, still under the equal likeliness assumption. Then calculate this probability. (Feel free to use Minitab or R for the calculation.)

$\Pr(X \geq 7) = \sum_{x=7}^{12} \binom{12}{x} (.2)^x (.8)^{12-x} \approx .0039$, as shown in the Minitab graph:



c) Is this probability small enough to cast doubt on the equal likeliness assumption? Explain.

Yes, this probability indicates that if the 5 players were equally likely to win each week, there's less than a 0.4% chance that I would win 7 or more of those 12 weeks. So, since I did win 7 of those 12 weeks, this suggests that my probability of winning is greater than .2.

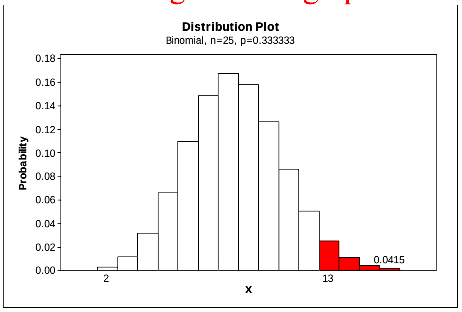
3. A cola discrimination study gives three cups of cola to each subject. Two of the cups have the same brand of cola, and the third cup has a different brand. The subject's task is to try to identify which cup has the different brand from the other two. Suppose that you conduct this study with a sample of 25 subjects. Let the random variable X denote the number of subjects who correctly identify the cup with the different brand.

a) If none of the subjects has any ability to discriminate, so they are all just guessing randomly among the three cups, what probability distribution would X have? (Specify its parameter values as well as its name.)

X ~ Binomial(n = 25, p = 1/3)

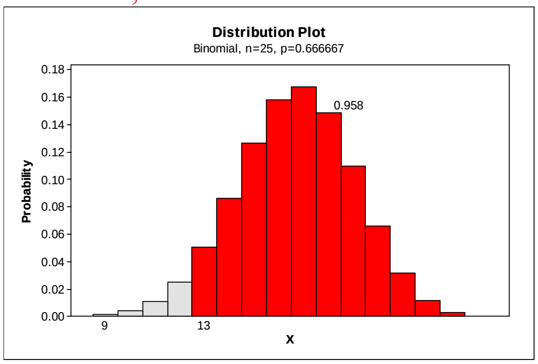
b) Determine how many subjects would have to make the correct identification in order for the probability of getting so many or more successes to be no greater than .05.

The following Minitab graph reveals that 13 is the necessary number correct:



c) Now suppose that subjects have some, but limited, ability to discriminate, to the point that the probability of successfully identifying the odd cola is 2/3. Determine the probability that the number of subjects who make the correct identification would be at least as large as your answer to b).

We now want Pr($Y \geq 13$), where $Y \sim$ Binomial($n = 25$, $p = 2/3$). Minitab reveals this probability to be .958, as shown below



d) Interpret what the probability in c) says.

If the subjects really can distinguish cola tastes, to the degree that they have a 2/3 probability of correctly identifying the odd cola, then there's about a 96% chance that 13 or more (out of 25 subjects) will correctly identify the odd cola, which will be enough to provide fairly convincing evidence that subjects do better than random guessing.