STAT 325 Introduction to Probability Models Spring 2010

HW12 due Mon May 3

Topics: Markov property, one-step transition probabilities, simulating Markov chains

1. Suppose you have \$3 but you desperately need to have \$5. You decide to play roulette to try to increase your \$3 into a \$5 fortune. You bet \$1 on a color and continue to do this until you have either \$5 or \$0. (Recall that your probability of winning \$1 with this bet is 18/38, and your probability of losing \$1 with this bet is 20/38.) The amount of money that you have after any spin of the wheel can be considered a Markov chain.

a) Identify the possible states of this Markov chain.

The possible states are \$0, \$1, \$2, \$3, \$4, and \$5.

b) Report the one-step transition probability matrix for this Markov chain. (Be sure to label the rows and columns for clarity.)

Note that when you have \$0, the probability is 1 that you will remain at \$0. Similarly, when you have \$5, the probability is 1 that you will remain at \$5. For all other starting values, you have a 18/38 probability of moving to one dollar less and a 20/38 probability of moving to one dollar more. The transition probability matrix is therefore:

	Move to \$0	Move to \$1	Move to \$2	Move to \$3	Move to \$4	Move to \$5
Start with \$0	1	0	0	0	0	0
Start with \$1	20/38	0	18/38	0	0	0
Start with \$2	0	20/38	0	18/38	0	0
Start with \$3	0	0	20/38	0	18/38	0
Start with \$4	0	0	0	20/38	0	18/38
Start with \$5	0	0	0	0	0	1

2. A simplified version of the Ehrenfest model of gas diffusion has six particles divided among two containers, A and B. At each step of the process, one particle is selected at random and is then moved to the other container. The number of particles in container A at any time can be considered a Markov chain.

a) Identify the possible states of this Markov chain.

The possible states are 0, 1, 2, 3, 4, 5, and 6.

b) Show the details of your calculations for determining the *middle row* of the one-step transition probability matrix.

When you start with 3 particles in container A, there's a 3/6 = .5 probability that one of them will be selected to move to container B, and a 3/6 = .5 probability that one of the 3 particles in container B will be selected to move to container A. Therefore, there's a .5 probability of moving to 2 particles in container A and a .5 probability of moving to 4 particles in container A.

c) Determine and report the one-step transition probability matrix.

The transition probability matrix is:

	Move to 0	Move to 1	Move to 2	Move to 3	Move to 4	Move to 5	Move to 6
Start with 0	0	1	0	0	0	0	0
Start with 1	1/6	0	5/6	0	0	0	0
Start with 2	0	2/6	0	4/6	0	0	0
Start with 3	0	0	3/6	0	3/6	0	0
Start with 4	0	0	0	4/6	0	2/6	0
Start with 5	0	0	0	0	5/6	0	1/6
Start with 6	0	0	0	0	0	1	0

3. Suppose that tomorrow's weather depends only on today's weather. Let's simplify by considering only three possibilities: sunny, cloudy, rainy. Suppose that:

- If it's sunny today, then it will be sunny tomorrow with probability .8, cloudy tomorrow with probability .2, and rainy tomorrow with probability 0.
- If it's cloudy today, then it will be sunny tomorrow with probability .4, cloudy tomorrow with probability .5, and rainy tomorrow with probability .1.
- If it's rainy today, then it will be sunny tomorrow with probability .3, cloudy tomorrow with probability .5, and rainy tomorrow with probability .2.

Suppose that the first day of the process is sunny, and subsequent days follow these probabilities.

a) Use R to simulate 100,000 days of this Markov chain weather process. (Feel free to use the pizzataco and/or pingpong code, available on the course website, as a model.) Submit your code.

Your code will vary a bit from mine, which is:

```
# start with N = number of repetitions
#
w = rep(NA, times = N)
rand = runif(N,0,1)
w[1] = "sunny"
for (i in 2:N) {
   if ((w[i-1] == "sunny") & (rand[i] < .8)) {w[i] = "sunny"}
   if ((w[i-1] == "sunny") & (rand[i] >= .8)) {w[i] = "cloudy"}
   if ((w[i-1] == "cloudy") & (rand[i] < .4)) {w[i] = "sunny"}
   if ((w[i-1] == "cloudy") & (rand[i] >= .4) & (rand[i] < .9)) {w[i] =
"cloudy"}
```

```
   if ((w[i-1] == "cloudy") & (rand[i] >= .9)) {w[i] = "rainy"}
   if ((w[i-1] == "rainy") & (rand[i] < .3)) {w[i] = "sunny"}
   if ((w[i-1] == "rainy") & (rand[i] >= .3) & (rand[i] < .8)) {w[i] =
"cloudy"}
   if ((w[i-1] == "rainy") & (rand[i] >= .8)) {w[i] = "rainy"}
}
table(w)
```

b) Use your simulation results to approximate the proportion of days that are sunny in the long run. Do the same for the proportion of days that are cloudy in the long run and the proportion of days that are rainy in the long run.

Your results will vary a bit. I obtained these approximate probabilities:

Sunny: .65972 Cloudy: .30267 Rainy: .03761

c) Produce 95% confidence intervals for the three probabilities that you approximated in b).

Based on my simulation results, a 95% CI for the steady-state probability of a sunny day is:

$.65972 \pm 1.96\sqrt{\dfrac{.65972 \times .34028}{100{,}000}}$, which is .65972 ± .00294, which is about (.657, .663).

Similarly, a 95% CI for the steady-state probability of a cloudy day is: $.30267 \pm 1.96\sqrt{\dfrac{.30267 \times .69733}{100{,}000}}$, which is .30267 ± .00285, which is about (.300, .306).

Similarly, a 95% CI for the steady-state probability of a rainy day is: $.03761 \pm 1.96\sqrt{\dfrac{.03761 \times .96239}{100{,}000}}$, which is .03761 ± .00118, which is about (.036, .039).