# STAT 325 Introduction to Probability Models Spring 2010

**HW15 due Mon May 10**

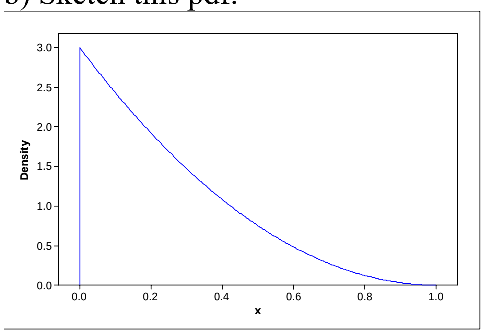
*Topics*: Probability density function, cumulative distribution function for continuous random variables

1. Suppose that the time at which a businessperson leaves for lunch (in hours after noon) is a continuous random variable X with probability density function $f(x) = c(1-x)^2$ for $0 < x < 1$, where $c$ is the appropriate constant, and $f(x) = 0$ otherwise.

a) Determine the value of $c$ for this to be a legitimate pdf.

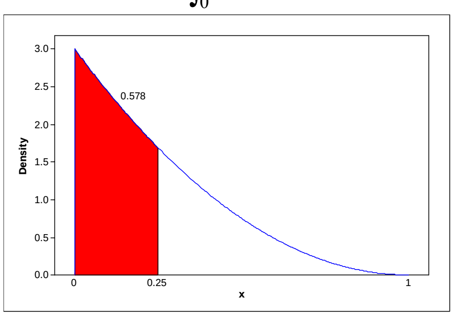
We need $\int_0^1 c(1-x)^2\,dx = 1$. The integral simplifies to $c/3$, so we need $c = 3$.

b) Sketch this pdf.




c) Determine the probability that this businessperson leaves for lunch before 12:15. Also shade the region corresponding to this probability in your pdf sketch.

$\Pr(X < .25) = \int_0^{.25} 3(1-x)^2\,dx = 1 - 27/64 = 37/64 \approx .578$




d) Determine the probability that this businessperson leaves for lunch at exactly 12:15.

$\Pr(X = .25) = 0$

e) Suppose that it's 12:30, and you notice that this businessperson has not yet left for lunch. Determine the (conditional) probability that he/she will leave before 12:45, given that he has not left by 12:30.

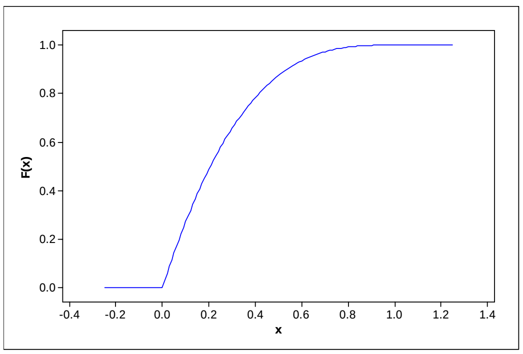
Pr(X < .75 | X > .5) = Pr[(X < .75) ∩ (X > .5)] / Pr(X > .5) = Pr(.5 < X < .75) / Pr(X > .5) = $\int_{.5}^{.75} 3(1-x)^2\,dx \;/\; \int_{.5}^{1} 3(1-x)^2\,dx$ = (7/64)/(1/8) = 7/8 = .875

f) Determine and sketch the cdf of X.

For $x \le 0$, F($x$) = 0

For $x \ge 1$, F($x$) = 1

For $0 < x < 1$, F($x$) = $\int_0^x 3(1-t)^2\,dt = 1 - (1 - x)^3$

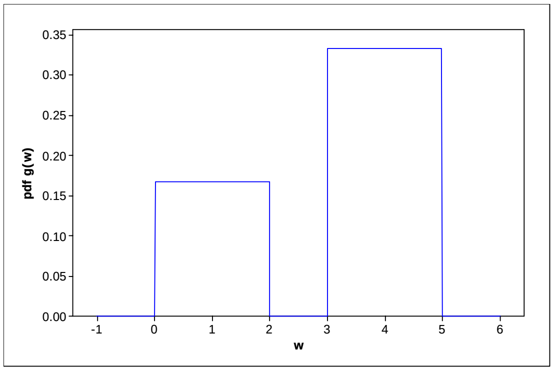



2. Suppose that the random variable W has a pdf of the following form: g($w$) = $c$ for $0 < w < 2$, g($w$) = 2$c$ for $3 < w < 5$, and g($w$) = 0 otherwise.

a) Determine the value of the constant $c$ that makes this a legitimate pdf. *Hint*: You may find it easier to use geometry than calculus.

The graph of this pdf consists of two rectangles, both with length 2, with the second rectangle twice as high as the first. The area of the first rectangle is 2$c$, and the area of the second rectangle is 4$c$. The total area is therefore 6$c$, so $c$ = 1/6 to make the total area = 1.

b) Produce a sketch of this pdf.




c) Determine Pr(1 < W < 4). Also shade the area under the pdf that corresponds to this probability.

The area of this region is 1/6 + 1/3 = ½ = .5.

d) Determine the cdf of W. *Hint*: Be sure to consider all real numbers as possible inputs to this function.

Let G($w$) = Pr(W ≤ $w$) be the cdf.

For values of $w \leq 0$, G($w$) = 0

For values with $0 < w < 2$, G($w$) = $w/6$

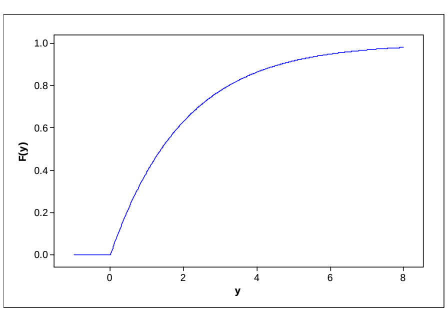
For values with $2 \leq w \leq 3$, G($w$) = 2/6 = 1/3 (the total area of the rectangle on the left)

For values with $3 < w < 5$, G($w$) = 1/3 + ($w$-3)/3 = ($w$-2)/3

For values of $w \geq 5$, G($w$) = 1

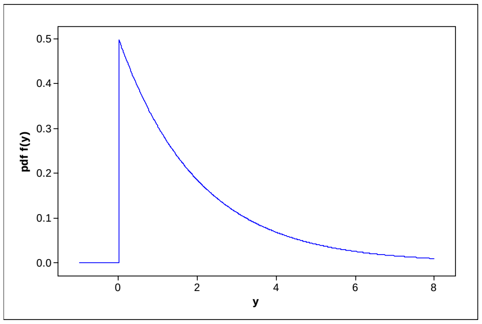
3. Suppose that the lifetime of a light bulb (in thousands of hours) is a continuous random variable Y with cumulative distribution function $F(y) = 1 - e^{-y/2}$ for $y > 0$, and $F(y) = 0$ for $y \leq 0$.

a) Sketch this cdf.




b) Determine and sketch the pdf of L.

The pdf is $f(y) = F'(y) = (1/2)e^{-y/2}$ for $y > 0$, $f(y) = 0$ for $y \leq 0$.




c) Use the cdf (not the pdf) to determine the probability that the light bulb lasts for more than 1 thousand hours.

$\Pr(Y > 1) = 1 - \Pr(Y \leq 1) = 1 - F(1) = 1 - (1 - e^{-1/2}) = e^{-1/2} \approx .607$

d) Given that the light bulb is still working after 2 thousand hours, determine the probability that it lasts for at least an additional 1 thousand hours.

$\Pr(Y > 3 \mid Y > 2) = \Pr[(Y > 3) \cap (Y > 2)] / \Pr(Y > 2) = \Pr(Y > 3) / \Pr(Y > 2) = e^{-3/2} / e^{-2/2} = e^{-1/2}$ $\approx .607$