STAT 325 Introduction to Probability Models Spring 2010

HW17 due Tues May 18

Topics: Exponential random variables, memoryless property

1. Suppose that the time (in hours) that Isabel spends on an untimed final exam follows an exponential distribution with mean 1.25 hours, and the time that Javier spends on the same exam follows an exponential distribution with mean 1.75 hours. Assume that their times are independent of each other.

a) Determine the probability that Isabel finishes in less than 1 hour.

Let I denote Isabel's time, so I ~ Expo(λ = 1/1.25 = 4/5 = .8). Pr(I < 1) = 1 – $e^{-.8}$ ≈ .551

b) Determine the probability that Javier finishes in less than 1 hour.

Let J denote Javier's time, so J ~ Expo(λ = 1/1.75 = 4/7 ≈ .571). Pr(J < 1) = 1 – $e^{-.571}$ ≈ .435

c) Determine the probability that both Isabel and Javier are still working on the exam after 1 hour. [Be sure to indicate which rules you use to combine probabilities.]

Using the multiplication rule for independent events, Pr[(I > 1) $\cap$ (J > 1)] = Pr(I > 1)×Pr(J > 1) = $(e^{-.8})(e^{-.571}) = e^{-1.371}$ ≈ .254

d) Determine the probability that at least one of these two people finishes in less than 1 hour. [Be sure to indicate which rules you use to combine probabilities.]

Using the complement rule, Pr[(I < 1) $\cup$ (J < 1)] = 1 - Pr[(I > 1) $\cap$ (J > 1)] ≈ 1 - .254 = .746

e) Determine the median value for Isabel (i.e., the value such that Isabel has probability .5 of finishing before or after that length if time).

We can find the median by setting Isabel's cdf equal to .5 and solving for *m*: $1 - e^{-.8m} = .5$, so $e^{-.8m} = .5$, so $-.8m = \ln(.5)$, so $m = -\ln(.5)/.8 \approx 0.866$ hours.

f) Determine the probability that Javier finishes before his mean time.

Pr(J < 1.75) = 1 – $e^{-.571(1.75)}$ = 1 - e^{-1} ≈ .632

g) Determine the conditional probability that Javier is still working after 3 hours, given that he is still working after 2 hours.

By the memoryless property, Pr(J > 3 | J > 2) = Pr(J > 1) = $e^{-.571}$ ≈ .565

h) Determine the conditional probability that Javier is still working after 3 hours, given that Isabel is still working after 2 hours.

Because I and J are independent, $\Pr(J > 3 \mid I > 2) = \Pr(J > 3) = e^{-.571(3)} = e^{-1.713} \approx .180$

2. Reconsider #1. Use an R simulation to approximate the probability that Isabel finishes the exam before Javier does. Submit your R code, and report your approximate probability, along with a 95% confidence interval for the exact probability.

The following R code performs this simulation, using N as the number of repetitions:

```
lambdai = 4/5
lambdaj = 4/7
itime = rexp(N,lambdai)
jtime = rexp(N,lambdaj)
isabeldonefirst = (itime < jtime)
approxprobs = table(isabeldonefirst)/N
approxprobs
```

Your results will vary slightly, but running this code for N = 1,000,000 repetitions gave me an approximate probability (that Isabel finishes first) of .583261. A 95% CI for the exact probability is then: $.583261 \pm 1.96\sqrt{\frac{.583261 \times .416739}{1{,}000{,}000}}$, which is .583261 ± .000966, which is (.582, .584), to 3 decimal places.