STAT 325 Introduction to Probability Models Spring 2010

HW 1 due Thur April 1

1. Another classic probability problem is called the collector's problem. For example, suppose that cereal boxes contain one prize each, with a total of 4 prizes to be collected. Also suppose that each box of cereal is equally likely to contain any one of the 4 prizes, and the particular prize that appears in one box has no bearing on the prize that appears in another box. You purchase one cereal box at a time, see which prize is in it, and continue buying one box at a time until you have the complete set of 4 prizes.

Describe how you could use four index cards to conduct a simulation analysis to approximate the probability that you would need to buy at least 10 boxes to find a complete set of prizes.

Mark each card to represent a different prize. Select a card at random, and note which prize it represents. Keep selecting one card at random, noting which prize it represents, until every prize has been selected. Record the number of selections required. Repeat this process a large number of times, and determine the proportion of repetitions for which at least selections have to be made.

2. A friend of mine commented that she was on a committee consisting of four male and two female faculty members. Two members of the committee were chosen to do some extra work, and it turned out that both females were chosen. My friend wondered how likely this result would have been if the selection (of two people to do the extra work) had been made at random (from among the six committee members).

Let's call the male members of the committee Abe, Ben, Chuck, and Don. Let's call the female members Ellen and Frannie.

a) List the sample space of all possible pairs of people who could have been chosen. (Just use initials to represent the people.)

S = {AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF}

b) Which outcomes comprise the event that Ellen is one of the people chosen?

{AE, BE, CE, DE, EF}

c) Are the events {Ellen is chosen} and {Frannie is chosen} mutually exclusive? Explain.

No, because the outcome EF belongs to both events.

d) Report the probability that both women would be selected, if the selection had been made at random.

This probability is 1/15 ≈ .0667, because there are 15 outcomes in S and only one outcome (EF) in which both women are selected.

e) Report the probability that at least one woman would be selected, if the selection had been made at random.

This event is the set {AE, AF, BE, BF, CE, CF, DE, DF, EF}, so its probability is 9/15 = 3/5 = .6.

3. The mathematical study of probability began with a gambling problem that famous mathematicians Pascal and Fermat corresponded about. The problem was to determine a fair way to divide a wager between two players when the game concluded before a winner could be decided. As an example, suppose that two people (Lorena and Manuel) bet on a sequence of coin tosses, with Lorena taking heads and Manuel taking tails. The game is supposed to end as soon as 5 heads or 5 tails are obtained. But suppose that they have to stop playing after the first 6 tosses, which result in 4 heads and 2 tails.

a) If the game had been able to continue, what are the minimum and maximum number of tosses that would have ensued? Explain.

The minimum number is 1, because the game would end if the next toss lands on heads. The maximum number is 3, because the game would end after 3 more tails or if any heads appears in those three tosses.

b) List the sample space of all possible outcomes of the remaining coin tosses. Use the notation H_i to mean that the i^{th} toss lands on heads and similarly for T_i. (So, one outcome in the sample space is T_1H_2.)

$S = \{H_1, T_1H_2, T_1T_2H_3, T_1T_2T_3\}$

c) Is it reasonable to assume that the outcomes in the sample space are equally likely? Explain.

No. An H or T outcomes on any one toss is equally likely, but some of these outcomes involve different numbers of tosses than others. More specifically, the H_1 outcome has probability ½, but the T_1H_2 outcome has probability ¼ because there are 4 equally likely outcomes for tossing a coin twice.

4. Consider again the sample space of 36 outcomes from rolling two fair six-sided dice.

a) Determine the probability that the first number rolled is larger than the second one.

There are 15 outcomes for which the first number is larger than the second: (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (6,5). The probability is therefore 15/36 = 5/12 ≈ .417.

b) Determine the probability that the two numbers differ by no more than 1.

There are 16 outcomes for which the two numbers differ by no more than 1: (1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4), (4,5), (5,4), (5,5), (5,6), (6,5), (6,6). The probability is therefore 16/36 = 4/9 ≈ .444.

c) Which pairs of the following events are mutually exclusive?

A = {sum is 7}
B = {sum is 11}
C = {1st number is 4)
D = {2nd number is 5}

A and B are mutually exclusive, because the sum cannot be both 7 and 11.
B and C are mutually exclusive, because the sum cannot be 11 when the 1st number is 4.