STAT 325 Introduction to Probability Models Spring 2010

HW20 due Wed May 26

Topics: Poisson process, inter-arrival times, Erlang distribution

1. Suppose that the occurrences of typographical errors ("typos") in a textbook follow a Poisson process with rate $\lambda = .175$ per page.

a) Determine the probability that a 10-page section of the textbook contains 2 or more typos.

$N(10) \sim \text{Poisson}(10\times.175 = 1.75)$, so $\Pr[N(10) \geq 2] = 1 - \Pr[N(10) = 0] - \Pr[N(10) = 1] = 1 - (e^{-1.75})(1.75^0)/0! - (e^{-1.75})(1.75^1)/1! \approx 1 - .174 - .304 = .522$

b) Let the random variable T be the "time" (measured in number of pages read) between "arrivals" of typos. Identify the probability distribution of T, and also report the mean and standard deviation of T.

$T \sim \text{Expo}(\lambda = .175)$, so $E(T) = 1/\lambda = 1/.175 \approx 5.714$ pages and $SD(T) = 1/\lambda = 1/.175 \approx 5.714$ pages

c) Determine the probability that you read more than 3.5 pages before finding a typo. Show how to calculate this probability using a Poisson distribution and also using an exponential distribution.

This is the probability of zero typos in 3.5 pages: $\Pr[N(3.5) = 0] = (e^{-3.5\times.175})[(3.5\times175)^0]/0! = e^{-.6125} \approx .542$

This is also the probability of the inter-arrival times exceeding 3.5: $\Pr(T > 3.5)$, where $T \sim \text{Expo}(\lambda = .175)$, which is $e^{-.6125} \approx .542$

d) Let the random variable W be the "time" (again measured in number of pages read) before you encounter 5 typos. Identify the probability distribution of W, both its name and its parameter value(s), and also report the mean and standard deviation of W.

$W \sim \text{Erlang}(k = 5, \lambda = .175)$, so $E(W) = k/\lambda = 5/.175 \approx 28.571$ pages and $SD(W) = \text{sqrt}(k)/\lambda = \text{sqrt}(5)/.175 \approx 12.778$ pages

e) Determine the probability that you read more than 10 pages before encountering 3 typos.

$\Pr[N(10) \leq 2] = (e^{-1.75})(1.75^0)/0! + (e^{-1.75})(1.75^1)/1! + (e^{-1.75})(1.75^2)/2! \approx .174 + .304 + .266 = .744$

2. Suppose that calls to 911 in SLO follow a Poisson process and that the expected time between successive calls is 1.75 hours.

a) Let N(1) represent the number of 911 calls in any 1-hour time period. State the probability distribution of N(1), both its name and its parameter value(s).

The inter-arrival time T has an exponential distribution with mean 1.75, so $\lambda = 1/1.75 = 4/7 \approx .571$, so $N(1) \sim \text{Poisson}(\lambda = 1/1.75 \approx .571)$.

b) Determine the probability that 2 or more calls to 911 occur in a 1-hour time period.

$\Pr[N(1) \geq 2] = 1 - \Pr[N(1) = 0] - \Pr[N(1) = 1] = 1 - (e^{-.571})(.571^0)/0! - (e^{-.571})(.571^1)/1! \approx 1 - .565 - .323 = .112$

c) Determine the probability that fewer than 90 minutes elapse between successive calls to 911.

Fewer than 90 minutes will elapse if more than one call occurs in those 90 minutes (1.5 hours):
$\Pr[N(1.5) > 0] = 1 - (e^{-1.5\times.571})[(1.5\times.571)^0]/0! = 1 - e^{-.857} \approx 1 - .424 = .576$

d) Determine the expected value and standard deviation of the number of 911 calls received by an operator who works an 8-hour shift.

$N(8) \sim \text{Poisson}(8\times4/7 \approx 4.571)$, so $E[N(8)] = 4.571$, $\text{Var}[N(8)] = 4.571$, $SD[N(8)] = \text{sqrt}(4.571) \approx 2.138$