STAT 325 Introduction to Probability Models Spring 2010

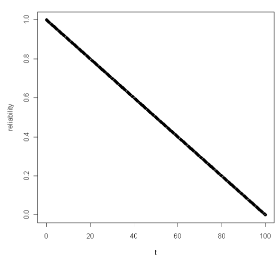
HW21 due Tues June 1

Topics: Reliability function, hazard rate

1. Suppose that a component has a lifetime (in hours) that is uniformly distributed on the interval (0, 100).

a) Determine and graph the reliability function of this component.

$R(t) = \Pr(T > t) = 1 - t/100$ for $0 < t < 100$, and $R(t) = 0$ for $t \geq 100$.

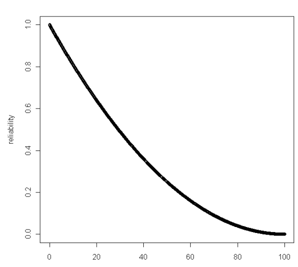


b) Calculate and interpret the reliability at time $t = 50$.

$R(50) = .5$, so the probability is .5 that the component will still be functioning after 50 hours.

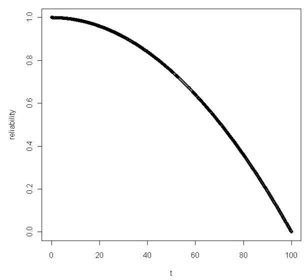
c) Now suppose that two such components, whose lifetimes are independent, are connected in series. Determine and graph the reliability function of this system.

Let T represent the lifetime of the system and T_i represent the lifetime of component i. The reliability is: $R(t) = \Pr(T > t) = \Pr[(T_1 > t) \cap (T_2 > t)] = R_1(t) \times R_2(t) = (1 - t/100)^2$.



d) Now suppose that two such components, whose lifetimes are independent, are connected in parallel. Determine and graph the reliability function of this system.

The reliability is: $R(t) = \Pr(T > t) = \Pr[(T_1 > t) \cup (T_2 > t)] = 1 - \Pr[(T_1 \leq t) \cap (T_2 \leq t)] = 1 - [1-R_1(t)]\times[1-R_2(t)] = 1 - t^2/10000$.



e) Calculate the reliability of both systems at time $t = 50$. Comment on how these compare to each other and to your answer to b).

Series: R(50) = .25

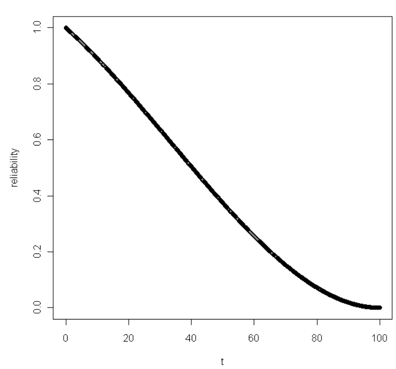
Parallel: R(50) = .75

Single-component: R(50) = .50

The parallel system has the highest reliability, and the series system has the lowest reliability.

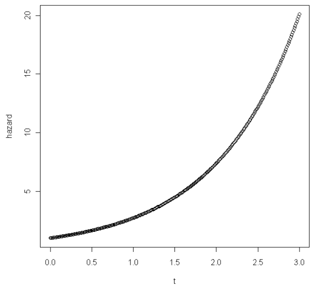
f) Now suppose that 3 components are connected in such a way that the system functions only if component 1 functions and either component 2 or component 3 functions. Determine the reliability function of this system. Be sure to show and explain the steps in your derivation.

$R(t) = \Pr(T > t) = \Pr\{(T_1 > t) \cap [(T_2 > t) \cup (T_3 > t)]\} = \Pr(T_1 > t) \times \Pr[(T_2 > t) \cup (T_3 > t)] = R_1(t) \times \{1 - [1-R_2(t)]\times[1-R_3(t)]\} = (1 - t/100) \times (1 - t^2/10000)$



2. Suppose that a system has hazard rate $h(t) = e^t$, $t > 0$.

a) Graph this hazard rate function, for $0 < t < 3$.

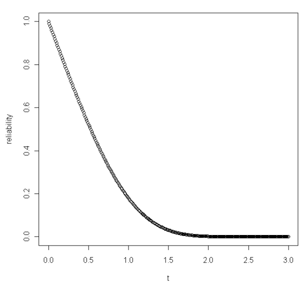


b) Is this hazard rate increasing or decreasing? Comment on what this reveals about the system.

The hazard rate is increasing, and the rate of increase itself is increasing. This reveals that the system is wearing out more and more quickly as time goes on.

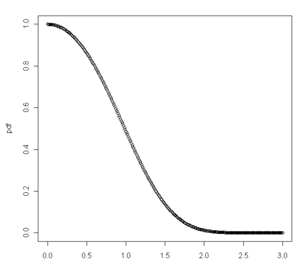
c) Determine and graph the reliability function of the system.

The reliability function is: R(t) = $e^{-\int_0^t h(s)ds} = e^{-\int_0^t e^s ds} = e^{1-e^t}$



d) Determine and graph the pdf of the lifetime T of the system.

The pdf is: f(t) = h(t) × R(t) = $e^t e^{1-e^t} = e^{1+t-e^t}$, for $t > 0$



e) Determine the probability that the system is still functioning at time $t = 1$. Indicate how you could use either the reliability function or the pdf to calculate this probability.

We could calculate this probability by integrating the pdf from 1 to infinity, or we can simply evaluate the reliability function at $t = 1$: R(1) = $e^{1-e} \approx .179$