STAT 325 Introduction to Probability Models Spring 2010

HW 2 due Mon April 5

Topics: Basic rules (complement, addition)

1. Suppose that 18% of Cal Poly students have received a speeding ticket and 37% have received a parking ticket.

Let E denote the event that a randomly selected student has received a speeding ticket, and let A denote the event that a randomly selected student has received a parking ticket.

a) Consider the event that a randomly selected Cal Poly student has received at least one ticket. Express this event with appropriate set notation and operations.

$E \cup A$

b) What is the largest possible value of the probability of the event that a randomly selected Cal Poly student has received at least one ticket? Explain.

The largest possible value for $E \cup A$ occurs when the events are mutually exclusive, in which case $Pr(E \cup A) = Pr(E) + Pr(A) = .18 + .37 = .55$.

c) Do the given probabilities allow for the possibility that E and A are mutually exclusive? Explain.

Yes, it's possible that E and A are mutually exclusive, because the probability of their union would still not exceed 1, as shown in b).

d) Is it reasonable to assume that E and A mutually exclusive in this context? Explain.

No. It's certainly possible, even likely, that some students have gotten both speeding and parking tickets.

e) Now consider the event that a randomly selected Cal Poly student has received both types of tickets. Express this event with appropriate set notation and operations.

$E \cap A$

f) What is the largest possible value of the probability of the event that a randomly selected Cal Poly student has received both types of tickets? Explain.

The largest possible value for $E \cap A$ is .18, because $Pr(E \cap A)$ cannot be larger than Pr(E) or Pr(A).

Now suppose further that 12% of Cal Poly students have received both a speeding ticket and a parking ticket.

g) Determine the probability that a randomly selected Cal Poly student has received at least one ticket.

By the addition rule, $Pr(E \cup A) = Pr(E) + Pr(A) – Pr(E \cap A) = .18 + .37 - .12 = .43$.

h) Determine the probability that a randomly selected Cal Poly student has received no tickets of either type. Also express this probability in appropriate set notation and operations.

Receiving no tickets of either type is the complement of receiving at least one ticket, so $Pr(E^c \cap A^c) = 1 – Pr(E \cup A) = 1 - .43 = .57$.

i) Determine the probability that a randomly selected Cal Poly student has received a ticket of one type but not the other type. Also express this probability in appropriate set notation and operations.

From the given information we can construct the following probability table:

	Speeding ticket	No speeding ticket	Total
Parking ticket	.12	.25	.37
No parking ticket	.06	.57	.63
Total	.18	.82	1.00

Now from the table we can see that $Pr[(E \cap A^c) \cup (E^c \cap A)] = .06 + .25 = .31$. We could also calculate this as $Pr(E \cup A) – Pr(E \cap A) = .43 - .12 = .31$.

2. Suppose that you encounter two traffic lights on your commute to school. You judge that the probability is .5 that the first light will be red when you get to it, .4 that the second light will be red, and .6 that *at least one* of the lights will be red.

a) Express these probabilities with proper notation and set operations.

Let R_1 be the event that the first light is red, and let R_2 be the event that the second light is red. We are told that $Pr(R_1) = .5$, $Pr(R_2) = .4$, and $Pr(R_1 \cup R_2) = .6$.

b) Determine the probability that *both* lights will be red. Show how your answer follows from a probability rule.

The addition rule says that $Pr(R_1 \cup R_2) = Pr(R_1) + Pr(R_2) – Pr(R_1 \cap R_2)$, so $.6 = .5 + .4 – Pr(R_1 \cap R_2)$, so $Pr(R_1 \cap R_2) = .5 + .4 - .6 = .3$.

c) Determine the probability that *neither* light will be red.

$Pr(R_1^c \cap R_2^c) = 1 – Pr(R_1 \cup R_2) = 1 - .6 = .4$.

d) Determine the probability that *exactly* one light will be red.

We can fill in this probability table:

	1st light red	1st light not red	Total
2nd light red	.3	.1	.4
2nd light not red	.2	.4	.6
Total	.5	.5	1.0

This table makes clear that $P[(R_1 \cap R_2^c) \cup (R_2 \cap R_1^c)] = .2 + .1 = .3$.

3. According to the 2006 *Statistical Abstract of the United States*, 36.1% of American households have a pet dog and 31.6% have a pet cat. Does it follow from this information that 67.7% have a pet dog *or* a pet cat? Explain/justify your answer, referring to at least one probability rule.

No. Some households have both a cat *and* a dog. The percentage with a cat *or* a dog will be 67.7% minus the percentage who have both, according to the addition rule.

4. a) Consider again rolling a pair of fair, six-sided dice. What is the probability that the two numbers obtained are consecutive integers (in either order)? Explain briefly.

Of the 36 outcomes in the sample space, 10 consist of consecutive integers: {(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4), (5,6), (6,5)}. The probability is therefore $10/36 = 5/18 \approx .278$.

b) Now suppose that I have six cards, one marked with number 1, one marked with 2, and so on through one marked with 6. You choose two cards at random, without replacing the first before you choose the second. Write out the sample space and determine the probability that the two numbers obtained are consecutive integers.

Now there are only 30 outcomes in the sample space, because (1,1), (2,2), (3,3), (4,4), (5,5), and (6,6) can no longer occur. There are still 10 outcomes that consist of consecutive integers, so the probability is now $10/30 = 1/3 \approx .333$.

c) Are the probabilities in a) and b) the same? If not, which is larger?

No, the probability in (b) is larger.

d) Now suppose that I have *n* cards, numbered with integers from 1 to *n*. You choose a card at random, replace it, and then choose another card at random. What is the probability (as a function of *n*) that the two numbers obtained are consecutive integers? Justify your answer.

There are n^2 outcomes in the sample space. If the first number is 1 or *n*, then there is only one outcome that gives consecutive integers. If the first number is anything else, then there are two outcomes that give consecutive integers. The number of outcomes with consecutive integers is therefore $1 + 1 + 2(n-2) = 2n-2 = 2(n-1)$. The probability of consecutive integers is therefore $2(n-1) / n^2$.

e) Repeat d) but under the condition that you do not replace the first card before choosing the second card.

The numerator is the same, because there are still $2(n-1)$ outcomes that consist of consecutive integers. But the denominator is now $n(n-1)$, because we lose the n outcomes that give the same number on both cards. The probability of consecutive integers is now $2(n-1) / n(n-1) = 2/n$.

f) Are the probabilities in d) and e) the same? If not, which is larger?

No, the probability in (e) is larger.

g) Are the functions in d) and e) increasing or decreasing as n increases? Justify your answer.

These are decreasing functions as n increases. This can be established by noting that the first derivative is negative.

h) Determine the limit of these functions as n approaches infinity. Justify your answer.

As n approaches infinity, the probability of obtaining consecutive integers approaches 0.