

STAT 325 Introduction to Probability Models Spring 2010

HW5 due Thur April 8

Topics: Conditional probability, multiplication rule, probability tree, law of total probability

1. As in HW2, suppose that 18% of Cal Poly students have received a speeding ticket, 37% have received a parking ticket, and 12% have received both types of tickets.

a) Determine the conditional probability that a randomly selected student has received a parking ticket, given that he/she has received a speeding ticket. (Also express this probability using appropriate notation, and do the same for other parts of this question.)

Let E denote the event that the student has received a speeding ticket, and let A denote the event that the student has received a parking ticket. Then $\Pr(A | E) = \Pr(A \cap E) / \Pr(E) = .12/.18 = 2/3 \approx .667$.

b) Determine the conditional probability that a randomly selected student has not received a speeding ticket, given that he/she has not received a parking ticket.

$\Pr(E^c | A^c) = \Pr(E^c \cap A^c) / \Pr(A^c) = .57 / .63 \approx .905$. Note: $\Pr(E^c \cap A^c) = 1 - \Pr(E \cup A) = 1 - [\Pr(E) + \Pr(A) - \Pr(A \cap E)] = 1 - (.18 + .37 - .12) = 1 - .43 = .57$.

c) Determine the conditional probability that a randomly selected student has received both types of tickets, given that he/she has received at least one type of ticket.

$\Pr(E \cap A | E \cup A) = \Pr(E \cap A) / \Pr(E \cup A)$, where the numerator is because the intersection of these two events is $E \cap A$. This becomes $.12/.43 \approx .279$.

2. Reconsider (yet again) the “random babies” problem of distributing four babies to four mothers at random.

a) Write out (again) the sample space of 24 outcomes.

$S = \{$ 1234 1243 1324 1342 1423 1432
2134 2143 2314 2341 2413 2432
3124 3142 3214 3241 3412 3421
4123 4132 4213 4231 4312 4321 $\}$

b) Report the probability that no mother receives the correct baby.

Nine of these outcomes result in no matches (in italics above), so this probability is $9/24 = 3/8 = .375$.

c) Given that the first mother receives the wrong baby, what is the (conditional) probability that no mother receives the correct baby?

The 18 outcomes in the bottom 3 rows of S have the first mother with the wrong baby. Nine of these have no matches, so this conditional probability is $9/18 = .5$.

d) Given that the first two mothers both receive the wrong baby, what is the (conditional) probability that no mother receives the correct baby?

This condition restricts the sample space to 14 outcomes:

2134 2143 2314 2341 2413 2432
3124 3142 3412 3421
4123 4132 4312 4321 }

Nine of these have no matches, so this conditional probability is $9/14 \approx .643$.

e) Given that the first three mothers all receive the wrong baby, what is the (conditional) probability that no mother receives the correct baby?

This condition restricts the sample space to 11 outcomes:

2143 2314 2341 2413
3124 3142 3412 3421
4123 4312 4321 }

Nine of these have no matches, so this conditional probability is $9/11 \approx .818$.

3. Suppose that there's a 30% chance of rain on Saturday. The probabilities for Sunday depend on what happens on Saturday. If it rains on Saturday, then there's an 80% chance of rain on Sunday. If it does not rain on Saturday, then there's a 10% chance of rain on Sunday.

a) Determine the probability that it rains on both Saturday and Sunday. Also express this event with appropriate notation, and indicate the rule used to calculate your answer.

Let R_1 denote rain on Saturday and R_2 denote rain on Sunday. Then the multiplication rule gives that $\Pr(R_1 \cap R_2) = \Pr(R_1) \times \Pr(R_2 | R_1) = (.3)(.8) = .24$.

b) Determine the probability that it does not rain on Sunday. Also express this event with appropriate notation, and indicate the rule used to calculate your answer.

According to the law of total probability, $\Pr(R_2^c) = \Pr(R_2^c | R_1) \times \Pr(R_1) + \Pr(R_2^c | R_1^c) \times \Pr(R_1^c) = (.2)(.3) + (.9)(.7) = .69$.

4. Consider a multiple choice exam with four options per question. Suppose that the probability is .6 that you know (with certainty) the answer to a randomly selected question. The probability is .3 that you'll eliminate (with certainty) 2 options and guess randomly between 2 options. The probability is .1 that you'll guess randomly among all 4 options. Determine the probability that

you answer a randomly selected question correctly. Also produce a tree diagram to represent the relevant probabilities that comprise your answer.

Let K be the event that you know the answer, G_2 that you guess between 2 options, and G_4 that you guess among all 4 options. Let C be the event that you answer the question correctly. The law of total probability establishes that:

$$\Pr(C) = \Pr(C | K) \times \Pr(K) + \Pr(C | G_2) \times \Pr(G_2) + \Pr(C | G_4) \times \Pr(G_4) = (1)(.6) + (.5)(.3) + (.25)(.1) = .775.$$