STAT 325 Introduction to Probability Models Spring 2010

HW7 due Thur Apr 15

Topics: Independence

1. a) Suppose again that 18% of Cal Poly students have received a speeding ticket, 37% have received a parking ticket, and 12% have received both. Suppose that a Cal Poly student is selected at random. Are the events {student has received a speeding ticket} and {student has received a parking ticket} independent? Justify your answer.

No, these events are not independent, because the product of their probabilities does not equal the probability of their intersection: $.18 \times .37 = .0666 \neq .12$.

b) I asked students in my STAT 221 class a few years ago to tell me their gender and how often they drink coffee. The results are displayed in the table below:

	Male	Female
Every day	7	15
Sometimes	12	10
(Almost) never	19	7

Suppose that one of these students is chosen at random. Are the events {student is male} and {student drinks coffee every day} independent? Justify your answer.

No, these events are not independent. One way to justify this is to note that Pr(drinks coffee every day) = $22/70 \approx .314$ but Pr(drinks coffee every day | male) = $7/38 \approx .184 \neq .314$.

c) Fill in the following table so the events {Dmitri has seen the film} and {Elena has seen the film} are independent:

	Dmitri has seen film	Dmitri has not seen film	Total
Elena has seen film			80
Elena has not seen film			
Total	30		100

We need $\Pr(D \cap E) = \Pr(D) \times \Pr(E)$. We know that $\Pr(D) = .3$ and $\Pr(E) = .8$, so we need $\Pr(D \cap E) = (.3)(.8) = .24$. Filling in the rest of the table gives:

	Dmitri has seen film	Dmitri has not seen film	Total
Elena has seen film	24	56	80
Elena has not seen film	6	14	20
Total	30	70	100

2. Recall the following scenario from HW1: Lorena and Manuel bet on a sequence of independent coin tosses, with Lorena taking heads and Manuel taking tails. The game is supposed to end as soon as 5 heads or 5 tails are obtained. But suppose that they have to stop playing after the first 6 tosses, which result in 4 heads and 2 tails. Determine the probability that

Lorena would win the game if they were able to continue at some later time. Also describe how you make use of the concept of independence in your solution.

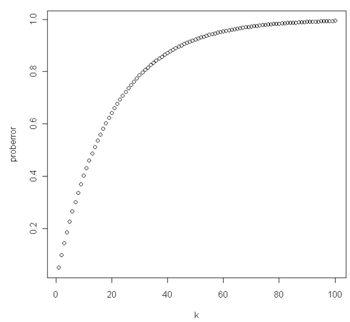
$\Pr(\text{Lorena wins}) = \Pr(H_1 \cup T_1H_2 \cup T_1T_2H_3) = \Pr(H_1) + \Pr(T_1H_2) + \Pr(T_1T_2H_3) = \Pr(H_1) + \Pr(T_1)\Pr(H_2) + \Pr(T_1)\Pr(T_2)\Pr(H_3) = .5 + .5\times.5 + .5\times.5\times.5$ (these multiplications are appropriate because the coin flips are independent) $= 7/8 = .875$.

3. Suppose that you perform *k* independent tests, with a .05 probability of making an error on each test.

a) Express the probability of making at least one error, as a function of *k*.

$\Pr(\text{error on at least one test}) = 1 - \Pr(\text{no errors}) = 1 - (.95)^k$

b) Produce a graph of this function, for values of *k* from 1 to 100.



c) How many tests must you perform before the probability of making at least one error exceeds .5?

We need $1 - (.95)^k > .5$, so $(.95)^k < .5$, so $k \ln(.95) < \ln(.5)$, so $k > \ln(.5) / \ln(.95) \approx 13.513$, so 14 or more tests are needed to produce a probability exceeding .5 of making at last one error.

4. Consider three basketball players: Annie makes 30% of her free throws, compared to 50% for Bella and 80% for Cassie. They play a contest in which the first player to make a free throw wins. The rules are that Annie shoots first. If she misses her shot, then Bella shoots next. If both miss, then Cassie shoots. If all three miss, the process begins again.

For each player, determine the probability that she wins this contest. Show the details of your calculations.

Let A_i denote the event that Annie makes her i^{th} shot, and similarly define B_i for Bella and C_i for Cassie.

The probability that Annie wins is:
$\Pr(A_1 \cup A_1^cB_1^cC_1^cA_2 \cup A_1^cB_1^cC_1^cA_2^cB_2^cC_2^cA_3 \cup \ldots)$

$= Pr(A_1) + Pr(A_1^c B_1^c C_1^c A_2) + Pr(A_1^c B_1^c C_1^c A_2^c B_2^c C_2^c A_3) + \ldots$ because these events are mutually exclusive

$= Pr(A_1) + Pr(A_1^c)Pr(B_1^c)Pr(C_1^c) + Pr(A_1^c)Pr(B_1^c)Pr(C_1^c)Pr(A_2^c)Pr(B_2^c)Pr(C_2^c)\ Pr(A_3) + \ldots$ because of independence

$= .3 + (.7)(.5)(.2)(.3) + (.7)(.5)(.2)\ (.7)(.5)(.2)(.3) + \ldots$

$= .3[1 + .07 + (.07)^2 + \ldots]$

$= .3/(1-.07)$

$= .3/.93$

$\approx .323$

Similarly, the probability that Bella wins is:

$(.7)(.5) + (.7)(.5)(.2)(.7)(.5) + (.7)(.5)(.2)(.7)(.5)(.2)(.7)(.5) + \ldots$

$= .35[1 + .07 + (.07)^2 + \ldots]$

$= .35/.93$

$\approx .376$

Similarly, the probability that Cassie wins is:

$(.7)(.5)(.8) + (.7)(.5)(.2)(.7)(.5)(.8) + (.7)(.5)(.2)(.7)(.5)(.2)(.7)(.5)(.8) + \ldots$

$= .28[1 + .07 + (.07)^2 + \ldots]$

$= .28/.93$

$\approx .301$