STAT 325 Introduction to Probability Models Spring 2010

HW8 due Mon Apr 19

Topics: Discrete random variable, probability mass function, cumulative distribution function

1. Recall from HW1 and HW7 the following scenario: Lorena and Manuel bet on a sequence of independent coin tosses, with Lorena taking heads and Manuel taking tails. The game is supposed to end as soon as 5 heads or 5 tails are obtained. But suppose that they have to stop playing after the first 6 tosses, which result in 4 heads and 2 tails. Let the random variable X denote the number of additional tosses that would be needed to complete their bet. Determine the pmf of X.

$\Pr(X = 1) = \Pr(H_1) = .5$

$\Pr(X = 2) = \Pr(T_1H_2) = \Pr(T_1)\Pr(H_2) = .5\times.5 = .25$

$\Pr(X = 3) = \Pr(T_1T_2H_3 \cup T_1T_2T_3) = \Pr(T_1T_2H_3) + \Pr(T_1T_2T_3) = \Pr(T_1)\Pr(T_2)\Pr(H_3) + \Pr(T_1)\Pr(T_2)\Pr(T_3) = .5\times.5\times.5 + .5\times.5\times.5 = .25$

The pmf of X is given in the following table:

x	1	2	3
$p(x)$	.5	.25	.25

2. Consider a random variable X with pmf:

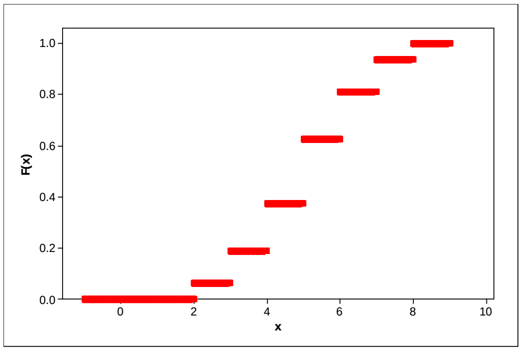
x	2	3	4	5	6	7	8
$p(x)$	1/16	1/8	3/16	1/4	3/16	1/8	1/16

a) Determine and graph the cdf of X.

Let $F(x)$ represent the cdf. This cdf can be expressed as:

$$F(x) = \begin{cases} 0 & x < 2 \\ 1/16 & 2 \le x < 3 \\ 1/16 + 1/8 = 3/16 & 3 \le x < 4 \\ 3/16 + 3/16 = 3/8 & 4 \le x < 5 \\ 3/8 + 1/4 = 5/8 & 5 \le x < 6 \\ 5/8 + 3/16 = 13/16 & 6 \le x < 7 \\ 13/16 + 1/8 = 15/16 & 7 \le x < 8 \\ 1 & x \ge 8 \end{cases}$$

A sketch of this cdf is:



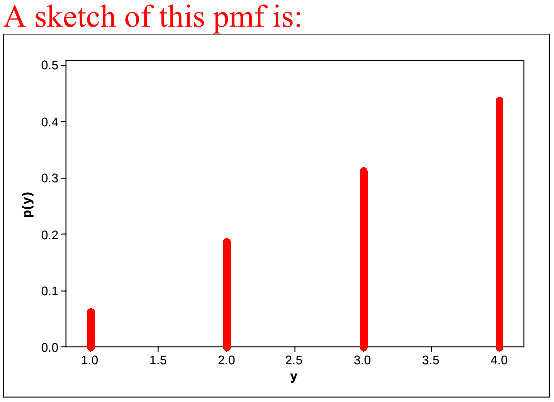
Now consider a random variable Y with cdf:

$$F(y) = \begin{cases} 0 & y < 1 \\ 1/16 & 1 \le y < 2 \\ 1/4 & 2 \le y < 3 \\ 9/16 & 3 \le y < 4 \\ 1 & y \ge 4 \end{cases}$$

b) Determine and sketch the pmf of Y.

The pmf is:
$p(1)$ = Pr(Y = 1) = 1/16
$p(2)$ = Pr(Y = 2) = 1/4 – 1/16 = 3/16
$p(3)$ = Pr(Y = 3) = 9/16 – 1/4 = 5/16
$p(4)$ = Pr(Y = 4) = 1 – 9/16 = 7/16

A sketch of this pmf is:



c) Both X and Y can be thought of as fairly simple functions applied to rolling two fair, *four*-sided dice. Try to determine what these functions are.

X = sum of the two dice rolls
Y = maximum of the two dice rolls

3. Suppose that I begin the day with 2 umbrellas at home and 1 umbrella in my office. If it's raining in the morning, then I'll take one umbrella from my home to the office. If it's raining in

the evening, then I'll take one umbrella from my office to my home. Suppose that the weather forecast says that there's a 30% chance of rain tomorrow morning. Additionally, the forecast says that if it's raining in the morning, then there's an 80% chance that it will also be raining in the evening. But if it's not raining in the morning, then there's only a 10% chance that it will be raining in the evening. Let the random variable X be the number of umbrellas that I have at my home at the end of the day.

a) List the possible values of X.

The possible values of X are 1, 2, and 3.

b) Determine the probabilities of these possible values. Show/justify your calculations. [*Hint*: First calculate probabilities for outcomes such as: rain in the morning and no rain in the evening.]

Let R_{am} denote the event that it is raining in the morning, and let R_{pm} denote the event that it is raining in the evening.

$\Pr(X = 1) = \Pr(R_{am} \cap R_{pm}^c) = \Pr(R_{am}) \times \Pr(R_{pm}^c \mid R_{am}) = (.3)(1\text{-}.8) = .06$

$\Pr(X = 3) = \Pr(R_{am}^c \cap R_{pm}) = \Pr(R_{am}^c) \times \Pr(R_{pm} \mid R_{am}^c) = (1\text{-}.3)(.1) = .07$

$\Pr(X = 2) = 1 - \Pr(X = 1) - \Pr(X = 3) = 1 - .06 - .07 = .87$