STAT 325 Introduction to Probability Models Spring 2012

Investigation 4: Random Rendezvous (assigned on Mon May 14, due on Tues May 22)

You may work with one partner, submitting one report with both names, provided that you both contribute substantially to the work. Word-processed reports are strongly preferred to hand-written ones. Please integrate computer output into your report.

Suppose two people (let's call them Cosette and Eponine) agree to meet for lunch at a certain restaurant, but both arrival times are random variables, independent of each other. If each person agrees to wait exactly fifteen minutes for the other before giving up and leaving, what is the probability that the two of them actually meet?

1. Uniform distributions

First suppose that the arrival time for each person, in minutes after noon, is *uniformly* distributed between 0 and 60.

a) Use R to simulate 10,000 days of the two people's arrival times. Produce a vector containing the absolute value of the difference in their arrival times. Also create a vector with an indicator of whether or not they successfully meet. Submit your R code. *R hint*: The command to generate random data from a uniform distribution is:

```
runif(number_reps, lower_endpoint, upper_endpoint)
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b) Produce and submit a histogram of the absolute value of the difference in their arrival times. Comment on what the graph reveals about this distribution.

c) Produce and submit a scatterplot of the pairs of arrival times, using different symbols/colors for the days in which they successfully meet and the days in which they do not meet. An example of R code for producing such a graph follows:

```
plot(time1, time2, type='n')
points(time1[meet=='TRUE'],time2[meet=='TRUE'],pch =16,col="green")
points(time1[meet=='FALSE'],time2[meet=='FALSE'],pch =17,col="red")
```

Comment on what the scatterplot reveals.

d) Increase the number of repetitions to 1,000,000 days. Report the approximate probability that the two people meet, along with a 95% confidence interval for the actual probability.

Because both distributions are uniform and independent of each other, exact probabilities can be calculated by determining the area of the region of interest as a fraction of the total area of the 60×60 square.

e) Use geometry to determine the exact probability that the two people meet. Include a sketch of the region of interest, and explain how you calculate the probability. Also comment on whether the exact probability is within your 95% confidence interval from d).

f) Now let *m* represent the number of minutes that both people agree to wait, where *m* can be any real number between 0 and 60. Use geometry to express the probability that they successfully meet as a function of *m*.

g) Produce (and submit) a graph of this function, and comment on its behavior.

h) Determine how long each person would have to agree to wait in order for this probability (of successfully meeting the other person) to equal .5. Then determine how long each person would have to agree to wait in order for this probability to equal .9.

2. Normal distributions

Now suppose that each person's arrival time, in minutes after noon, follows a normal distribution with mean 30 and SD 10.

i) Report the probability distribution of the *difference* (not absolute difference) in the arrival times of Cosette and Eponine.

j) Express the probability that the two people meet in terms of Φ, the cdf of a standard normal distribution.

k) Use appropriate normal probability calculations to determine the probability that the two people meet. (Here and below, indicate the probability that you are calculating but then feel free to use software to calculate it.)

l) Now let *m* represent the number of minutes that both people agree to wait, where *m* can be any real number. Determine the value of *m* so the probability of meeting is .9.

m) Now suppose that both people are able to be more precise about their arrival times, reducing their SDs to 5 minutes. Re-answer i), j), and k), comment on how your answers change, and explain why this makes sense intuitively.

Now suppose that instead of both people agreeing to wait for 15 minutes, Cosette agrees to wait for 20 minutes but Eponine can only wait for 10 minutes. (Change their SDs back to 10 minutes for each person.)

n) Determine the probability that they successfully meet, and comment on how this has changed from k).

Now suppose that Cosette changes his arrival time distribution to be normally distributed with mean 20 minutes after noon and SD 10 minutes, while Eponine's arrival time is normally distributed with mean 30 and SD 10. Go back to assuming that they both wait for 15 minutes.

o) Determine the probability that Cosette arrives first.

p) Determine the probability that they successfully meet.

3. Exponential distributions

Now suppose that the two people's arrival times are independent *exponential* distributions with mean 15 minutes after noon. Continue to assume that each waits for 15 minutes.

q) Use R to simulate 10,000 days of the two people's arrival times. Submit your R code. Also produce and submit a scatterplot of the pairs of arrival times, using different symbols/colors for whether or not the people successfully meet. Comment on what this scatterplot reveals about the distribution of arrival times.

r) Increase the number of repetitions to 1,000,000 days. Report the approximate probability that the two people meet, along with a 95% confidence interval.

s) Now suppose that Cosette changes her mean to 10 minutes after noon, and Eponine changes her mean to 20 minutes after noon. Again use R to simulate 10,000 days of the two people's arrival times. Submit your R code. Again produce and submit a scatterplot of the pairs of arrival times, using different symbols/colors for whether or not the people successfully meet. Describe how things have changed.

t) Increase the number of repetitions to 1,000,000 days. Report the approximate probability that the two people meet, and comment on how it has changed.

u) Based on your simulation of 1,000,000 days, report the approximate probability that Cosette arrives first, along with a 95% confidence interval.