Example 1-1: Random Babies
Suppose that four mothers give birth to baby boys at the same hospital on the same evening. As a very, very sick joke (do not try this at home!), the hospital staff decides to return the babies to the mothers at random! How likely is it (i.e., what is the probability) that at least one mother receives the correct baby?

This is a classic probability problem known as the *matching problem*. If you prefer to think of a different context, suppose that four college students bump into each other and drop their cell phones; then each student picks up one of the four cell phones at random.

We’ll take two approaches to answering this question:
1) An approximate analysis via simulation
2) An exact analysis via enumeration

- A process is **random** if individual outcomes are uncertain but there is a regular distribution of outcomes in a large number of repetitions.
- The **probability** of any outcome in a random process can be interpreted as the proportion (relative frequency) of times that the outcome would occur in a very large number of repetitions.
- A probability can be approximated by **simulating** (artificially re-creating) the random process a large number of times and determining the relative frequency of occurrences.

a) Describe how you could use four index cards to simulate this random process and approximate the probability of interest.

b) Use four index cards to conduct 5 repetitions of this random process. For each repetition, record the number of mothers who receive the correct baby.

c) Combine your simulation results with those of your classmates. Report the approximate probability that at least one mother receives the correct baby.
d) Now we turn to technology in order to perform this simulation more quickly and efficiently. We will use an applet found at: [www.rossmanchance.com/applets/randomBabies/Babies.html](http://www.rossmanchance.com/applets/randomBabies/Babies.html). (Be forewarned that this applet contains rather graphic images that reveal where babies come from!) Conduct 1000 repetitions of this simulation. Report the approximate probability of interest.

- Conducting more repetitions in a simulation analysis generally produces more accurate approximations of probabilities.
- A (rough) rule-of-thumb is that the approximate probability will likely fall within $\pm \frac{1}{\sqrt{N}}$ of the actual probability, where $N$ represents the number of repetitions.

e) Calculate and interpret the value of $\frac{1}{\sqrt{N}}$ for your applet simulation.

Can we use a more mathematical analysis to calculate this probability exactly? Yes, and we will soon, but first we need to learn some basic probability ideas.

**Example 1-2: Ice Cream Prices**

Suppose that you have only 50 cents in your pocket and you want to buy an ice cream cone. The owner of the ice cream shop offers a random price determined as follows: You roll a pair of fair, six-sided dice, and the price is the larger number followed by the smaller number (in cents). What is the probability that you’ll be able to afford the ice cream cone?

a) Let’s use the software package R to simulate this random process with 10,000 repetitions:

```r
d1 = sample(1:6, 10000, replace=TRUE)
d2 = sample(1:6, 10000, replace=TRUE)
price = 10*pmax(d1,d2) + pmin(d1,d2)
afford=(price<=50)
sum(afford)/10000
```

Report the approximate probability and also a rough estimate of its accuracy.

Conducting an exact mathematical analysis involves listing all possible outcomes of the random process.

- The **sample space** $(S)$ of a random process is a set consisting of all possible outcomes.
- An **event** is a subset of the sample space (often denoted by a capital letter).
- If the outcomes are equally likely, then the probability of the event is the number of outcomes in the event divided by the total number of outcomes in the sample space.
- Choosing an object “at random” means that the possible objects are equally likely.
• Two events are **mutually exclusive** if they have no outcomes in common (i.e., if their intersection is the empty set).

c) List the 36 outcomes in the sample space for rolling a pair of fair, six-sided dice.

d) Circle the outcomes that comprise the event that you can afford the ice cream cone.

e) Determine the (exact) probability that you can afford the ice cream cone.

f) Is the exact probability close to our simulation approximations?

**Example 1-3: Random Babies (cont.)**
Recall that we simulated the process of giving four babies back to their mothers at random. The sample space of all possible outcomes can be listed as follows:

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a) How many outcomes are in this sample space?

b) Count how many of these outcomes produce at least one mother with the correct baby.

c) Report the (exact) probability that at least one mother receives the correct baby. Is this close to our approximations from simulation?