A random variable is one whose value is determined by some chance mechanism. It can be thought of as a function that takes an outcome in a sample space as input and then gives a real number as output.

- With a discrete random variable, the set of possible values is either finite or countably infinite.
- With a continuous random variable, the probability is spread continuously over intervals of real numbers.

Random variables are often denoted with capital letters, toward the end of the alphabet.
- Lower case letters denote possible values of random variables.

Example 7-1: Matching Babies (cont.)
Recall the “random babies” process in which four babies are returned to their mothers at random. We represented the sample space of possible outcomes as:

```
1234 1243 1324 1342 1423 1432
2134 2143 2314 2341 2413 2431
3124 3142 3214 3241 3412 3421
4123 4132 4213 4231 4312 4321
```

Let the random variable $X = \text{number of mothers who get the correct baby}$.

a) For each of the 24 outcomes in the sample space above, determine the numerical value that the random variable $X$ assigns to that outcome.

b) List all possible values of the random variable $X$. Then report the probability for each of those possible values.

c) Write out the pmf of the random variable $X$ in this case.

The probability mass function (pmf) of a discrete random variable $X$ is a function that assigns a probability to each possible value of $X$.

- The pmf of $X$ is denoted by $p(x)$, where $p(x) = \text{Pr}(X = x)$.
  - $p(x) \geq 0$ for all $x$
  - $\sum_x p(x) = 1$

c) Write out the pmf of the random variable $X$ in this case.
d) For the following values $x$, report $\Pr(X \leq x)$. [Note that this is not asking for $\Pr(X = x)$.]

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr(X \leq x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e) For the following real numbers $x$, report $\Pr(X \leq x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>1.5</th>
<th>$\pi$ (3.14159 ...)</th>
<th>56.2897</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr(X \leq x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- The cumulative distribution function (cdf) of a (discrete or continuous) random variable $X$ is a function that takes any real number as input and outputs the probability that the random variable is less than or equal to the input value.
  - The cdf is typically denoted by a capital letter and is defined for all real numbers $x$ by: $F(x) = \Pr(X \leq x)$.
  - The cdf is a non-decreasing function.
  - The cdf approaches 0 as the input approaches negative infinity.
  - The cdf approaches 1 as the input approaches positive infinity.

f) Write out the cdf for this random variable $X$.


g) Create a graph of this cdf. How would you describe such a graph/function?

- The cdf of a discrete random variable is a step function.
  - The steps occur at the possible values of the random variable.
  - The height of a particular step corresponds to the probability of that value.
Example 7-2: Rolling Dice (cont.)

a) Consider a random variable X with pmf given by: 
\[
p(x) = \begin{cases} 
5/12 & x = -1 \\
1/6 & x = 0 \\
5/12 & x = 1 \\
0 & \text{otherwise}
\end{cases}
\]

Determine and graph the cdf of X.

b) Consider a random variable Y with cdf given by: 
\[
F(y) = \begin{cases} 
0 & y < 0 \\
1/6 & 0 \leq y < 1 \\
4/9 & 1 \leq y < 2 \\
2/3 & 2 \leq y < 3 \\
5/6 & 3 \leq y < 4 \\
17/18 & 4 \leq y < 5 \\
1 & y \geq 5
\end{cases}
\]

Determine and graph the pmf of Y.
Example 7-3: Solitaire
Suppose that every night I play Solitaire on my computer until I win for the first time. Let’s suppose that my probability of winning any one game is 1/9 and that the results of the games are independent. Let the random variable $Z =$ number of games that I play in order to achieve my first win.

a) What are the possible values of $Z$?

b) Determine and graph the pmf of $Z$.

c) Verify that the pdf sums to 1. Also indicate the calculus tool needed to do this.

d) Determine and graph the cdf of $Z$.

Example 7-4: World Series
Suppose that two evenly matched teams play a sequence of games until one of them has won four games. Assume that each team has a .5 probability of winning each game, independently from game to game. Let the random variable $G$ represent the number of games that are played. Determine the pmf of $G$. 