Let the continuous random variable $X$ represent the lifetime (in thousands of hours) for a particular electrical component, and suppose that $X$ follows an exponential distribution with mean (expected value) of 2.5 (thousand hours).

a) Determine the probability that this component lasts for more than 2 thousand hours.

Note that $X \sim \text{Expo}(\lambda = 1/2.5 = 0.4)$.

Also remember that we derived an expression that $\Pr(X > x) = e^{-\lambda x}$.

So, $\Pr(X > 2) = e^{-0.4 \times 2} = e^{-0.8} \approx 0.449$

b) Given that the component has already lasted for 10 thousand hours, determine the probability that it lasts for more than additional 2 thousand hours. Also cite the name of the property that is relevant here.

By the memoryless property of the exponential distribution,

$$\Pr(X > 12 \mid X > 10) = \Pr(X > 2) = e^{-0.8} \approx 0.449$$

Now consider an electrical system with 5 such independent components connected in series, meaning that the system stops working as soon as any one of the components stops working. Let the continuous random variable $T$ represent the lifetime of the system.

c) Identify the probability distribution of $T$. (Be sure to state its parameter value(s) as well as the name of the distribution.)

Notice that $T = \min\{X_1, X_2, X_3, X_4, X_5\}$,

because the system’s lifetime is the same as the lifetime of whichever component is the first to fail.

Therefore, by the result we derived in class, $T \sim \text{Expo}(\lambda = 0.4 \times 5 = 2)$.

d) Determine the probability that the system lasts for more than 2 thousand hours.

$$\Pr(T > 2) = e^{-2(2)} = e^{-4} \approx 0.018$$
Another way to see that is that for the system to last for more than 2 thousand hours, all of the components must last for more than 2 thousand hours.

By the multiplication rule for independent events,

$$\Pr(X_1 > 2, X_2 > 2, X_3 > 2, X_4 > 2, X_5 > 2) = (e^{-0.8})^5 = e^{-4} \approx .018.$$