Consider a stochastic process \( X(t) \) given by \( X(t) = Acos(2\pi t) + (B+1)sin(2\pi t) \), where \( A \) and \( B \) are independent random variables with \( E(A) = E(B) = 0 \) and \( Var(A) = Var(B) = 1 \).

a) Determine the mean function of \( X(t) \).

\[
E[X(t)] = E[Acos(2\pi t) + (B+1)sin(2\pi t)]
\]

(note that the random variables are \( A \) and \( B \))

\[
= E(A)\times cos(2\pi t) + E(B+1)\times sin(2\pi t)
\]

\[
= 0\times cos(2\pi t) + 1\times sin(2\pi t)
\]

\[
= sin(2\pi t)
\]

b) Determine the autocorrelation function of \( X(t) \).

\[
R_X(t, \tau) = E[X(t)X(t+\tau)]
\]

\[
= E\{(Acos(2\pi t) + (B+1)sin(2\pi t))[Acos(2\pi t+2\pi\tau) + (B+1)sin(2\pi t+2\pi\tau)]\}
\]

\[
= E(A^2)\times cos(2\pi t)\times cos(2\pi t+2\pi\tau) + E[A(B+1)]\times cos(2\pi t)\times sin(2\pi t+2\pi\tau)
\]

\[
+ E[A(B+1)]\times cos(2\pi t+2\pi\tau)\times sin(2\pi t) + E[(B+1)^2]\times sin(2\pi t)\times sin(2\pi t+2\pi\tau)
\]

The middle two terms are zero because \( A \) and \( B \) are independent and so \( E[A(B+1)] = E(A)E(B+1) = 0\times 1 = 0 \)

This leaves:

\[
R_X(t, \tau) = E(A^2)\times cos(2\pi t)\times cos(2\pi t+2\pi\tau) + E[(B^2+2B+1)]\times sin(2\pi t)\times sin(2\pi t+2\pi\tau)
\]

Now \( E(A^2) = Var(A) + [E(A)]^2 = 1 \) and similarly \( E(B^2) = 1 \), so this becomes:

\[
R_X(t, \tau) = cos(2\pi t)\times cos(2\pi t+2\pi\tau) + 2\times sin(2\pi t)\times sin(2\pi t+2\pi\tau)
\]

c) Determine the autocovariance function of \( X(t) \).
The autocovariance function $C_X(t, \tau) = R_X(t, \tau) - E[X(t)]E[X(t+\tau)]$

$= \cos(2\pi t)\cos(2\pi t+2\pi \tau) + 2\sin(2\pi t)\sin(2\pi t+2\pi \tau) - \sin(2\pi t)\sin(2\pi t+2\pi \tau)$

$= \cos(2\pi t)\cos(2\pi t+2\pi \tau) + \sin(2\pi t)\sin(2\pi t+2\pi \tau)$

d) Is $X(t)$ a wide-sense stationary process? Explain why or why not.

No, because both the mean function and autocorrelation function depend on $t$ as well as $\tau$. 