Suppose that Sofia is trying to break the piñata at her birthday party. The first time that she swings at it, there is a 70% chance that it will break. If the piñata doesn’t break on a particular swing, the chance that it will break on the next swing goes up by 10 percentage points. (So, if it doesn’t break on the first swing, the chance that it will break on the second swing is 80%, and so on.) Sofia continues to swing until she breaks the piñata, and then she does not swing again. Consider the random variable $X = \text{total number of swings required to break the piñata.}$

Determine the probability mass function of $X$. (List the possible values of $X$ and their probabilities.)

Let $S_i$ mean that Sofia is successful on her $i^{\text{th}}$ attempt and $F_i$ mean that she fails on her $i^{\text{th}}$ attempt.

Pr($X = 1$) = Pr($S_1$) = .7
Pr($X = 2$) = Pr($F_1S_2$) = Pr($F_1$)$\times$Pr($S_2|F_1$) = (.3)(.8) = .24
Pr($X = 3$) = Pr($F_1F_2S_3$) = Pr($F_1$)$\times$Pr($F_2|F_1$)$\times$Pr($S_3|F_1F_2$) = (.3)(.2)(.9) = .054
Pr($X = 4$) = Pr($F_1F_2F_3S_4$) = Pr($F_1$)$\times$Pr($F_2|F_1$)$\times$Pr($F_3|F_1F_2$)$\times$Pr($S_4|F_1F_2F_3$) = (.3)(.2)(.1)(1) = .006

So, the pmf of $X$ is:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>.7</td>
<td>.24</td>
<td>.054</td>
<td>.006</td>
</tr>
</tbody>
</table>