

Stat 426 Assignment 2 (due Wed, Jan 23)

Topics: Binomial, Geometric, Hypergeometric, Poisson, Multinomial Gamma, Exponential Distributions

1. Suppose that ten college professors meet to play basketball at lunchtime. To choose teams, they want to divide themselves into two teams of five at random. Consider two strategies for doing this: A) All ten flip a fair coin. If they get a 5-5 split of heads/tails, that determines the two teams. If they do not get a 5-5 split, they all flip again, and repeat until they do get a 5-5 split. B) The first nine to arrive all flip a fair coin. If they get a 5-4 or 4-5 split, then the tenth player joins the team with 4, and that determines the teams. If they do not get a 5-4 or 4-5 split, then those nine flip again, and repeat until they do get a 5-4 or 4-5 split.

- a) Determine the probability that one set of flips will suffice to determine the teams with strategy A.
- b) Determine the probability that one set of flips will suffice to determine the teams with strategy B.
- c) With whichever strategy has the higher probability of determining the teams with one set of flips, determine the probability that the teams will be determined in three or fewer sets of flips.

2. Reconsider the previous problem. Now suppose that the goal is to randomly split n players (where n is an even number, so $n = 2k$ for some integer k) into two groups of k each. Strategy A is for all n players to flip a fair coin and see if an even split (k heads and k tails) results. Strategy B is for $(n-1)$ players to flip a fair coin and see if k or $k-1$ flips result in heads.

- a) Express the probability that Strategy A succeeds in one set of flips, as a function of k .
- b) Express the probability that Strategy B succeeds in one set of flips, as a function of k .
- c) Investigate whether one strategy gives a higher probability than the other of succeeding on one set of flips. State and prove a result about this question.

3. D&S, page 262, #8. [*Hint:* Start with the definition of conditional probability.]

4. A study published in the journal *Neurology* examined whether botox is helpful for reducing pain among patients who suffer from chronic low back pain. Thirty-one subjects participated in the study. They were randomly assigned to one of two treatment groups: 15 received the botox itself, and the other 16 received a placebo of normal saline. The subjects' pain levels were evaluated at the beginning of the study and again after three weeks and after eight weeks.

Naturally, the researchers were looking for evidence that botox was more effective than the placebo for reducing back pain. The results of the study after eight weeks were that 9 of the 15 subjects who received botox experienced a substantial reduction in pain, compared to 2 of the 16 subjects in the placebo group.

- a) Suppose that botox was actually not effective and that the 11 subjects who felt pain relief would have done so no matter which group they had been assigned to. Let the random variable X be the number of "successes" (those experiencing pain reduction) assigned at random to the botox group. Describe the probability distribution of X . [Be sure to state the parameter values as well as the name of the distribution.]
- b) Determine the probability of obtaining a result as extreme as the researchers found, if in fact botox was not effective.

c) Based on the probability in b), do the experimental data provide much evidence that botox is effective? Explain.

5. Suppose that the number of typographical errors on a randomly selected page of a particular textbook follows a Poisson distribution with parameter $\lambda = 0.2$.

a) Determine the probability that a randomly selected page would have 0 errors.

b) Determine the probability that a randomly selected page would have at least 2 errors.

Now suppose that you select 10 pages at random.

c) Determine the probability that you find a total of at least 2 errors on these pages. [Be sure to justify whatever probability distribution you use.]

d) Determine the probability that at least 1 page contains at least two errors. [Be sure to justify whatever probability distribution you use.]

6. Suppose that the number of colds that Lorena experiences in a year follows a Poisson distribution with parameter $\lambda = 0.8$. Lorena decides to use a zinc supplement that is effective for 70% of the population. For individuals for whom the zinc supplement is effective, the Poisson parameter is reduced to $\lambda = 0.4$. Individuals for whom the zinc supplement is not effective are neither helped nor hurt by it (i.e., λ remains 0.8).

a) Suppose that Lorena makes it through the year experiencing 0 colds. Determine the (updated) probability that the zinc supplement is effective for Lorena.

b) Repeat (a) supposing that Lorena experiences one cold in the year.

7. a) Suppose that eight people are chosen at random. If each person is equally likely to have any one of the four blood types (A, B, AB, O), what is the probability that two people of each type are selected?

b) Now suppose that eight white Californians are selected at random and that the probabilities of having the four blood types are as reported on page 310 of D&S. Determine the probability that two people of each type are selected.

8. Suppose that X has an exponential distribution with parameter β .

a) Determine the median of X .

b) Comment on whether the mean is greater than or less than the median, and explain why this makes sense.

9. D&S, page 302, #13, 14

10. a) Suppose that the time (in hours) required for Brad to finish a take-home exam follows an exponential distribution with mean 3 hours, and the time required for Tom to finish the exam follows an exponential distribution with mean 5 hours. Suppose further that these times are independent. Determine the probability that Brad finishes the exam before Tom does.

b) Repeat a) in the more general case that Brad's mean is μ_B and Tom's mean is μ_T , still assuming independent exponential distributions.