Stat 426 Assignment 3 (due Tues, Jan 29)
Topics: Beta, Normal, Bivariate Normal Distributions; Central Limit Theorem

1. Suppose that a random variable X has a Beta distribution has mean .6 and standard deviation .1.
a) Determine the values of the parameters α and β.
b) Produce a sketch of the pdf. [Feel free to use software.]
c) Calculate Pr(X > .5). [Feel free to use software.]

2. D&S, page 281, #14

3. a) Determine the lower quartile (LQ) and upper quartile (UQ) of a standard normal distribution. [Hint: The lower quartile is defined to be $\Phi^{-1}(.25)$. In other words, $\Phi(LQ) = .25$.]
b) A common rule identifies outliers as values more than 1.5×(UQ-LQ) below LQ or more than 1.5×(UQ-LQ) above UQ. According to this rule, what proportion of values from a standard normal distribution are outliers?

4. Suppose that the drying time for a certain type of paint under specified test conditions is known to be normally distributed with mean $\mu = 75$ minutes and standard deviation $\sigma = 4$ minutes. Suppose that chemists have devised a new additive that is hoped will reduce the mean drying time (without changing the standard deviation). Suppose that a test is conducted to measure the drying time for a test specimen, and suppose that company executives decide that they will be convinced that the additive is effective only if the drying time on this specimen is less than 70 minutes.
a) If the additive actually has no effect at all on the drying time, what is the probability that the company executives will mistakenly conclude that it is effective (because the drying time turns out to be less than 70 minutes)?
b) Now suppose that the additive really is effective and that it reduces the mean drying time to $\mu = 67$ minutes, without changing the standard deviation of $\sigma = 4$ minutes. What is the probability that this test will fail to convince the executives that the additive is effective (because the drying time turns out to be 70 minutes or more), even though the additive actually is effective?
c) If you want alter the cut-off value from 70 in order to reduce the error probability in a) to .05, what cut-off value should you choose?
d) Suppose again that the additive really is effective and that it reduces the mean drying time to $\mu = 67$ minutes, without changing the standard deviation of $\sigma = 4$ minutes. Using this new cut-off value in c), what is the probability that that the test will fail to convince the executives that the additive is effective, even though it actually is?
e) How does the probability in d) compare to that in b)? Explain why this makes sense.

5. Let V = log(W), and suppose that V has a normal distribution with mean μ and standard deviation σ. Then W is said to have a lognormal distribution with parameters μ and σ. [Note that log represents natural log here.]
a) Determine the pdf of W. [*Hint*: You might want to refer back to section 3.8.]
b) Suppose that $\mu = 2$ and $\sigma = 4$. Determine Pr(W > 1), Pr(W > 100), and Pr(W > 10,000).

6. D&S, page 318, #2, 3, 4

7. D&S, page 291, #8. Be sure to explain the justification for any approximation that you use.

8. In its simplest form, betting on a color in the game roulette provides a 18/38 probability of winning $1 and a 20/38 probability of losing $1.
a) Use the CLT to approximate the probability that your net winnings will be negative if you play 50 times.
b) Use the CLT to approximate the probability that your net winnings will be negative if you play 500 times.
c) How does these probabilities compare? Explain why this makes sense.