

Stat 426 Assignment 4 (due Tues, Feb 12)

Topics: Prior and Posterior Distributions, Conjugate Prior Distributions, Bayes Estimators

1. . D&S, page 334, #3. Also answer:

b) Determine the posterior mean of λ and compare it to the prior mean of λ . Explain why it makes sense that the mean changed as it did in light of the observed sample data.

2. Suppose that X_1, X_2, X_3 form a random sample from the (discrete) uniform distribution on the integers $1, 2, \dots, N$. Suppose that the prior distribution of N is: $\pi(6) = .4, \pi(8) = .2, \pi(10) = .4$.

a) Determine the posterior distribution of N , given that $X_1 = 5$. [*Hint*: Notice that the parameter space consists of only three values.]

b) Determine the posterior distribution of N , given that $X_1 = 5$ and $X_2 = 7$. [*Hint*: You can use the posterior distribution from a) as the prior distribution for the $X_2 = 7$ observation. Also: think before you calculate!]

3. In a study reported in the November 2007 issue of *Nature*, researchers investigated whether infants take into account an individual's actions towards others in evaluating that individual as appealing or aversive, perhaps laying for the foundation for social interaction (Hamlin, Wynn, and Bloom, 2007). In one component of the study, 10-month-old infants were shown a "climber" character (a piece of wood with "google" eyes glued onto it) that could not make it up a hill in two tries. Then they were alternately shown two scenarios for the climber's next try, one where the climber was pushed to the top of the hill by another character ("helper") and one where the climber was pushed back down the hill by another character ("hinderer"). The infant was alternately shown these two scenarios several times. Then the child was presented with both pieces of wood (the helper and the hinderer) and asked to pick one to play with. The researchers found that the 14 of the 16 infants chose the helper over the hinderer. Let θ be the probability that an infant selects the helper toy. Suppose that the prior distribution of θ is uniform on the interval $(0,1)$.

a) Determine the posterior distribution of θ .

b) Produce a sketch of the prior and posterior distributions, on the same scale.

c) Determine the posterior mean and standard deviation of θ .

d) Determine the posterior probability that $\theta > .5$.

e) Determine the posterior probability that $\theta > .75$.

4. Reconsider the study and data from #3. Re-answer questions a)-e), using a prior distribution for θ that is $\text{Beta}(4,2)$.

5. Reconsider the study and data from #3. Now analyze the data from a classical rather than a Bayesian perspective.

a) Suppose that $\theta = .5$ (i.e., that infants have no preference and so are equally likely to select the helper or hinderer toy). Determine the probability that a random sample of 16 infants would produce 14 or more who choose the helper toy.

b) Now suppose that $\theta = .75$ (i.e., that infants are three times as likely to select the helper over the hinderer toy). Determine the probability that a random sample of 16 infants would produce 14 or more who choose the helper toy.

c) Comment on how these p-values in a) and b) compare to the posterior probabilities in #3 and #4.

6. D&S, page 345, #6. [*Hint*: Let X be the number of defects in a 100-foot roll of this tape; assume that X has a Poisson distribution with parameter θ .]

Also answer:

b) Determine the posterior mean of θ and compare it to the prior mean of θ . Explain why it makes sense that the mean changed as it did in light of the sample data.

7. D&S, page 345, #11

8. Reconsider the class example in which conditional on θ , X_1, X_2, \dots, X_n form a random sample from an $\text{Exponential}(\theta)$ distribution. We took the prior distribution of θ to be a $\text{Gamma}(3,1)$ distribution, and we found the posterior distribution of θ given $X_1 = 2$ to be a $\text{Gamma}(4,3)$ distribution. Now suppose that we observe $X_2 = 3$.

a) Determine the posterior distribution of θ , given that $X_1 = 2$ and $X_2 = 3$.

b) Determine the marginal distribution of X_3 , given that $X_1 = 2$ and $X_2 = 3$.

c) Determine $\Pr(X_3 > 1)$, given that $X_1 = 2$ and $X_2 = 3$.

d) Determine $E(X_3)$, given that $X_1 = 2$ and $X_2 = 3$.

9. D&S, page 346, #21

10. D&S, page 354, #12