Stat 427 Homework Assignment 1 (due Wednesday, April 9)
Topics: Testing Simple Hypotheses, Neyman-Pearson Lemma

1. Reconsider the class example in which X has a Beta(θ, 1) distribution. Now consider testing the simple hypotheses \( H_0: \theta = 1 \) vs. \( H_1: \theta = k \), where \( k > 1 \).
a) Let \( k = 4 \). Determine the test procedure that minimizes the sum of the probabilities of type I and type II errors.
b) Determine the probability of type I error and the probability of type II error with this procedure in a), still using \( k = 4 \).
c) Determine the test procedure that minimizes the sum of the probabilities of type I and type II errors for a general value of \( k \), where \( k > 1 \).

2. Reconsider the class example in which \( X_1, X_2, \ldots, X_n \) are a random sample of size \( n \) from an exponential distribution with parameter \( \theta \) (and therefore mean \( 1/\theta \)). Consider testing the hypotheses \( H_0: \theta = 1 \) vs. \( H_1: \theta = 2 \). Let \( a \) and \( b \) be positive constants.
a) Determine the test procedure that minimizes \( a \times \alpha + b \times \beta \).
b) Now consider testing \( H_0: \theta = \theta_1 \) vs. \( H_1: \theta = \theta_2 \), where \( \theta_1 \) and \( \theta_2 \) are positive constants with \( \theta_2 > \theta_1 \). Determine the test procedure that minimizes \( a \times \alpha + b \times \beta \) for these hypotheses.

3. Suppose that a single observation \( X \) is to be drawn from an unknown distribution and that the following simple hypotheses are to be tested:

\[
H_0: \text{X has a uniform distribution on the interval (-2.5, 2.5)}
\]
\[
H_1: \text{X has a standard normal distribution}
\]

a) Determine the test procedure that minimizes the sum of the probabilities of type I and type II errors. Be sure to specify what the test procedure stipulates for all real values of \( x \). [\text{Hint: You might obtain some intuition for this situation by graphing the two pdfs on the same scale, perhaps with Minitab’s Graph> Probability Distribution Plot> Two Distributions command.}]
b) Calculate the probability of type I error and the probability of type II error with this procedure.

4. Reconsider the class example in which \( X_1, X_2, \ldots, X_n \) are a random sample of size \( n \) from a normal distribution with mean \( \theta \) and standard deviation 1. Let \( c \) be a positive constant, and consider testing the hypotheses \( H_0: \theta = 0 \) vs. \( H_1: \theta = c \).
a) Use the Neyman-Pearson Lemma to determine the optimal test procedure with a significance level of .05.
b) Derive an expression for the probability of type II error for this test procedure.
c) Describe the effect of increasing the value of \( c \) on the rejection region and on the probability of type II error for this test procedure.

5. D&S, page 469, #4

6. D&S, page 470, #8, 9

7. D&S, page 470, #10

8. D&S, page 470, #11