Stat 427 Homework Assignment 1 (due Wednesday, April 5)
Topics: Accuracy of Simulations, Testing Simple Hypotheses, Neyman-Pearson Lemma

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. from an exponential distribution with mean β.
a) Determine the median of this distribution, in terms of the mean β. (You did this twice in Stat 426.)
b) Use simulation to investigate whether the sample median $\tilde{X}$ is an unbiased estimator of the population median (that you determined in a). Choose two different values of n and two different values of β. For each of these four pairs of choices, generate 10,000 random samples, calculate the 10,000 simulated sample medians, create and submit a histogram of the 10,000 simulated sample medians, and determine a 95% confidence interval for $E(\tilde{X})$. Also write a paragraph summarizing your results.

2. Recall that in Investigation 7 of Stat 426, you used simulation to investigate whether the conventional *t*-confidence interval worked well (i.e., was robust) when the population follows an exponential distribution. Now you will assess the accuracy of such a simulation as well as the performance of the *t*-procedure.
a) Take the parameter value to be $\beta = 1$. Choose three different sample sizes to give a reasonable sense of how the procedure performs with a small, moderate, and large sample. For each sample size, generate 10,000 random samples, calculate the 10,000 simulated 90% *t*-confidence intervals, determine how many intervals succeed in capturing the population mean, and determine a 95% confidence interval for the (true) probability that the 90% *t*-procedure successfully captures the population mean. Also write a paragraph summarizing your results.
b) Based on your results in a), determine how many simulated samples N would be necessary to estimate the true capture probability to within +.005 with 95% confidence. Answer this question separately for each of your sample sizes from a).

3. Reconsider the class example in which $X_1, X_2, \ldots, X_n$ are a random sample of size n from an exponential distribution with parameter θ (and therefore mean $1/\theta$). Consider testing the hypotheses H_0: $\theta = 1$ vs. H_1: $\theta = 2$. Let a and b be positive constants.
a) Determine the test procedure that minimizes $a \times \alpha(\delta) + b \times \beta(\delta)$.
b) Now consider testing H_0: $\theta = \theta_1$ vs. H_1: $\theta = \theta_2$, where θ_1 and θ_2 are positive constants with $\theta_2 > \theta_1$. Determine the test procedure that minimizes $a \times \alpha(\delta) + b \times \beta(\delta)$ for these hypotheses.

4. Reconsider the class example in which $X_1, X_2, \ldots, X_n$ are a random sample of size n from a normal distribution with mean θ and standard deviation 1. Let c be a positive constant, and consider testing the hypotheses H_0: $\theta = 0$ vs. H_1: $\theta = c$.
a) Use the Neyman-Pearson Lemma to determine the optimal test procedure with a significance level of .05.
b) Describe the effect of increasing the value of c on the rejection region for this optimal test procedure.

5. D&S, page 469, #2
6. D&S, page 469, #4
7. D&S, page 470, #7.
8. D&S, page 470, #10.