Stat 427 Homework Assignment 2 (due Wednesday, April 12)

Topics: Monotone Likelihood Ratio, Uniformly Most Powerful Tests, Two-Sided Alternatives

1. Reconsider #4 from HW1, in which $X_1, X_2, \ldots, X_n$ are a random sample of size *n* from a normal distribution with mean θ and standard deviation 1. Let *c* be a positive constant, and consider testing the hypotheses H_0: $\theta = 0$ vs. H_1: $\theta = c$.
a) Derive an expression for the probability of type II error for the optimal test procedure with a significance level of .05.
b) Construct a graph of this probability as a function of *c*.
c) Comment on how this probability changes as *c* increases, and explain why this makes sense intuitively.

2. Let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. from a Beta(θ,1) distribution. Find a statistic $T = r(Y_1, Y_2, \ldots, Y_n)$ so that the distribution has monotone likelihood ratio (MLR) in this statistic. [Be sure to show that all of the conditions for MLR are satisfied by this statistic.]

3. Reconsider the class example in which $X_1, X_2, \ldots, X_n$ are a random sample of size *n* from a normal distribution with mean μ and known variance σ^2, and you want to test H_0: $\mu \leq \mu_0$ vs. H_a: $\mu > \mu_0$. We determined the UMP test procedure for a given significance level α_0, and we determined an expression for the power function of this UMP test. Determine how (if at all) the rejection region changes as:
a) the sample size *n* increases
b) the significance level α_0 increases
c) the population variance σ^2 increases
d) the hypothesized value of the mean μ_0 increases
Justify all of your answers mathematically, and also explain why each makes sense intuitively.

4. D&S, page 485, #6, 7. [Use $n = 10$ and $\alpha_0 = .05$ for your sketch.]

5. D&S, page 478, #8, 9.

6. Reconsider the class example in which $X_1, X_2, \ldots, X_n$ are a random sample of size *n* from a normal distribution with (unknown) mean μ and (known) standard deviation σ. Continue to consider testing H_0: $\mu = \mu_0$ vs. H_1: $\mu \neq \mu_0$.
a) Determine the α=.05 test procedure that uses $\alpha_1 = .025$ and $\alpha_2 = .025$.
b) Determine the α=.05 test procedure that uses $\alpha_1 = .02$ and $\alpha_2 = .03$.
c) Determine the α=.05 test procedure that uses $\alpha_1 = .04$ and $\alpha_2 = .01$.

Now let n=25, σ=12, and μ_0=100.
d) Specify the rejection region of each test procedure (from parts a, b, c) in this case.
e) Use Minitab (or another computer program) to graph the power functions of the three test procedures on the same graph. Also report the values of the power function for each procedure at μ=95, at μ=98, at μ=100, at μ=102, and at μ=105.
f) For every possible value of μ (not just those listed in part a), identify which of these three test procedures has the highest power.
g) For each of these three test procedures, indicate whether it is an unbiased test, and explain your answers.

7. Reconsider the previous exercise and the test procedure from part a). Continue to let σ=12 and μ_0=100, but now let the sample size be n.
a) Derive an expression for the power of the test at μ=98, as a function of the sample size n.
b) Create a graph of the power as a function of n. [*Hint*: You may want to use Minitab or another computer program.]
c) Determine the smallest value of n for which the power at μ=98 is at least .80.
d) Determine the smallest value of n for which the power at μ=99 is at least .80. Is this larger or smaller than your answer to c)? Explain why this makes sense intuitively.

8. D&S, page 485, #10.