Stat 427 Homework Assignment 3 (due Thursday, April 20)
Topics: *t*-tests, Non-central *t*-distribution, Comparing Two Normal Means, F-distribution

1. Reconsider the class example concerning points scored in NBA games. Recall that we were testing the hypotheses H_0: $\mu \leq 183.2$ vs. H_a: $\mu > 183.2$ at the $\alpha_0 = .05$ level, based on a sample of $n = 25$ games.
a) Suppose that the population mean μ is actually equal to $183.2 + \sigma/4$, where σ is the population standard deviation. [Note that this says that the population mean is actually one-quarter of a standard deviation higher than the hypothesized value.] Determine the power of the *t*-test against this alternative.
b) Determine the power if the sample size is increased to $n = 50$.
c) Repeat, for a sample size of $n = 100$.
d) Write a Minitab macro that calculates the power as a function of n. Also produce a graph of the power as a function of n. Start with $n = 2$, and go as large as necessary for the power to exceed .99. [Submit your macro, including an explanation of every step, and also the graph.]
e) Determine how large n must be for this power to exceed .9.

2. Reconsider the class example concerning the golden ratio. Recall that we were testing the hypotheses H_0: $\mu = .618$ vs. H_a: $\mu \neq .618$ at the $\alpha_0 = .05$ level, based on a sample of $n = 20$ rectangles.
a) Suppose that the population mean is actually $\mu = .65$ and the population standard deviation is actually $\sigma = .1$. Determine the probability of type II error in this case.
b) Determine and graph the power of this test, as a function of μ. [Include enough values for μ so that the power gets to be at least .99 on both sides.]

3. The sample data in the Minitab worksheet MarriageAges.mtw are the ages of 24 couples who applied for marriage licenses in Cumberland County, Pennsylvania. Consider testing whether husbands tend to be older than their wives on average.
a) State the appropriate null and alternative hypotheses in symbols and in words.
b) Specify the rejection region for the appropriate *t*-test at the α=.05 significance level.
c) Calculate the value of the *t*-test statistic based on the sample data. Would you reject the null hypothesis at the α=.05 significance level?
d) Determine the smallest significance level for which these sample data would lead to rejecting the null hypothesis.

4. Let $X_1, X_2, \ldots, X_n$ be i.i.d. from a normal distribution with mean μ_x and standard deviation σ, and let $Y_1, Y_2, \ldots, Y_n$ be i.i.d., and independent of the X_i's, from a normal distribution with mean μ_y and the same standard deviation σ. [Note that the sample sizes and population standard deviations are the same for the two groups.] Assume that σ is known but μ_x and μ_y are not. Consider testing the hypotheses H_0: $\mu_x \leq \mu_y$ vs. H_a: $\mu_x > \mu_y$ at the α_0 significance level, and consider a test procedure that rejects H_0 when $\overline{X} - \overline{Y} \geq c$, where c is an appropriate constant.
a) Express c as a function of n, σ, and α_0. [Be sure to justify every step in your derivation.]
b) Determine the value of c when $n = 20$, $\sigma = 10$, and $\alpha_0 = .01$.
c) Let $\chi = \mu_x - \mu_y$. Determine and graph the power of the test in b) as a function of χ.
d) How far apart must the population means μ_x and μ_y be in order for this test to have power .75?

5. Let $X_1, X_2, \ldots, X_m$ be a random sample of size m from a normal distribution with mean μ_x and standard deviation σ, and let $Y_1, Y_2, \ldots, Y_n$ be an independent random sample of size n from a normal distribution with mean μ_y and the same standard deviation σ. We have defined the pooled sample variance to be $\frac{\sum_{i=1}^{m}(X_i-\overline{X})^2+\sum_{i=1}^{n}(Y_i-\overline{Y})^2}{m+n-2}$. As usual, define each group's sample variance as $\frac{\sum_{i=1}^{m}(X_i-\overline{X})^2}{m-1}$ and $\frac{\sum_{i=1}^{n}(Y_i-\overline{Y})^2}{n-1}$.

a) Show that the pooled sample variance can be written as a weighted average of the two group's sample variances, and determine what those weights are.

b) Researchers measured the body temperatures for a sample of 65 healthy males and 65 healthy females. The sample standard deviation of the males' temperatures is 0.743 degrees Fahrenheit, and the sample standard deviation of the females' temperatures is 0.699 degrees Fahrenheit. Use your result to determine the pooled sample standard deviation of the body temperatures.

c) Is the pooled sample standard deviation equal to the average of the two groups' sample standard deviations when the two groups have the same sample size? Explain.

6. Reconsider the "body temperatures" study described above.

a) Conduct a two-sided F-test of whether the population variances of body temperatures differ between men and women. Report the hypotheses, the rejection region for the α=.05 significance level, the observed value of the test statistic, your test decision for the α=.05 significance level, and the p-value of the test.

b) The sample mean body temperatures are 98.105 for the females and 98.394 for the males. Conduct a two-sided pooled *t*-test of whether the population means of body temperatures differ between men and women. Report the hypotheses, the rejection region for the α=.05 significance level, the observed value of the test statistic, your test decision for the α=.05 significance level, and the p-value of the test.

c) Use a pooled *t*-procedure to determine a 95% confidence interval for the difference in population mean body temperatures between men and women.

7. Reconsider the "French fries" data, which reports the number of fries in a small bag for a sample of 10 McDonald's bags and 5 Burger King bags. The sample data are:

McDonald's:	35	39	43	52	53	55	34	37	45	40
Burger King:	51	55	52	43	44					

Determine a 90% confidence interval for the difference in population means using three methods:

a) pooled t

b) unpooled t with complicated (Welch) degrees of freedom

c) unpooled t with conservative (min of n_x-1 and n_y-1) degrees of freedom

d) Comment on how the three intervals differ, and explain why this makes sense.

8. D&S, page 530, #2, #3.