

Stat 427 Homework Assignment 5 (due Monday, May 15)

Topics: Simpson's Paradox, Sign and Rank Tests, Distribution of Sample Median

1. The following tables classify 326 murder cases in which the defendant was convicted and eligible to receive the death penalty. The three variables are the defendant's race, the victim's race, and whether or not the defendant was sentenced to death.

White defendant	Death	No death	Black defendant	Death	No death
White victim	19	132	White victim	11	52
Black victim	0	9	Black victim	6	97

(a) Create a two-way table of defendant's race by death penalty.

(b) Perform the calculations to show that Simpson's paradox holds:

- A higher percentage of white than of black defendants are sentenced to death.
- Among those who killed white victims, a higher percentage of black than of white defendants are sentenced to death.
- Among those who killed black victims, a higher percentage of black than of white defendants are sentenced to death.

(c) Basing your reasoning on the data, explain why the paradox occurs, in language that a judge could understand.

2. Reconsider the hospital example, where hospital A had the higher recovery rate for both types of patients (poor condition, fair condition) and yet had the lower recovery rate overall. Continue to define the events: $A = \{\text{patient went to hospital A}\}$, $B = \{\text{patient went to hospital B}\}$, $F = \{\text{patient was in fair condition}\}$, $O = \{\text{patient was in poor condition}\}$, $R = \{\text{patient recovered}\}$. (Note that $B = A^c$ and $O = F^c$.) But now consider the situation in general, not the specific data analyzed previously.

a) Show that if $P(F|A) = P(F|B)$, then the paradox can not occur. In other words, show that if $P(F|A) = P(F|B)$, and if $P(R|AF) > P(R|BF)$ and if $P(R|AO) > P(R|BO)$, then $P(R|A) > P(R|B)$.

b) Explain in common language what the result in a) says.

3. Reconsider the cell phone study from Exam 1. Recall that reaction times (in milliseconds) were recorded for a group of 16 subjects under two different driving conditions: once while talking on a cell phone and once while listening to an audiobook. The data are in the Minitab worksheet `CellPhone.mtw`.

a) Perform a sign test on these data. State the hypotheses, and explain how the test statistic and p-value are calculated. Also indicate what test decision you would make at the $\alpha = .01$ level, and interpret your conclusion in context.

b) Repeat (a) for a signed rank test.

c) Repeat (a) for a paired t -test.

d) Comment on how the p-value changes across these three procedures and on whether your conclusion changes for any of these analyses.

4. Researchers studied the effects of sleep deprivation by conducting a study in which some students were randomly assigned to be denied sleep on the first night of the study and other students were randomly assigned to sleep as long as they wanted on that night. All students were allowed unrestricted sleep on the second and third nights of the study.

The variable measured was the improvement in score on a visual task between performing the task on the first day and on the fourth day of the study (so a negative score indicates a decrease in performance). The data are in the Minitab worksheet `SleepDeprivation.mtw`.

a) Conduct a rank sum (Wilcoxon-Mann-Whitney) test of whether the data provide strong evidence that the sleep-deprived group tended to have lower improvements than the unrestricted sleep group. Show the details of the calculation of the test statistic by hand, and use the normal approximation to calculate the (approximate) p-value.

b) Conduct a pooled t -test of whether the data provide strong evidence that the sleep-deprived group has a lower population mean improvement than the unrestricted sleep group. Report the sample means, sample standard deviations, test statistic, and p-value. [You may use Minitab.]

c) Comment on how the p-values of the rank sum and pooled t -tests compare for these data.

5. a) Create a hypothetical example of 15 couple's marriage ages with the property that a sign test produces a (one-sided) p-value less than .01 but a paired t -test produces a (one-sided) p-value greater than .15. Submit a list of the 15 differences, the test statistics, and p-values. Also write a sentence or two explaining the reasoning behind your choice of example. [Hints: You are welcome to use Minitab. Your example should not be identical to anyone else's.]

b) Repeat a) with the property that a sign test produces a (one-sided) p-value greater than .15 but a signed rank test produces a (one-sided) p-value of less than .01.

6. D&S, page 594, #11

7. Let X_1, X_2, \dots, X_n constitute a random sample from a uniform distribution on the interval $(\theta - 1, \theta + 1)$. Consider the sample mean \bar{X} and the sample median \tilde{X} as estimators of the parameter θ .

a) Determine the MSE of each estimator. [Use the asymptotic result for the sample median.]

b) Which estimator has the smaller MSE?

c) Calculate the ratio of the MSE of the sample mean to the MSE of the sample median. Does this ratio change as the sample size n increases? Explain.

8. Let X_1, X_2, \dots, X_n constitute a random sample from a contaminated normal distribution, where the primary distribution is normal with mean θ and variance σ^2 and the contamination distribution is normal with mean θ and variance $10\sigma^2$. Each X_i has probability ε of coming from the contamination distribution and probability $(1-\varepsilon)$ of coming from the primary distribution. Determine the values of ε for which the sample mean has a smaller variance than the sample median, using the asymptotic result for the variance of the sample median.