Stat 427 Homework Assignment 7 (due Thursday, June 2)
Topics: Bootstrap Distribution, Bootstrap Confidence Interval, Decision Theory, Utility, Bayes Action, Expected Value of Sample Information

1. Reconsider the data on height-to-width ratios of beaded rectangles used by the Shoshoni American Indians to decorate their leather goods (`shoshoni.mtw`.) Let μ represent the population mean of these ratios.
a) Determine a 95% *t*-interval for μ.
b) Generate 10,000 bootstrap samples, and calculate the mean of each. Produce, submit, and comment on a histogram of the resulting bootstrap distribution of the sample mean.
c) Report the bootstrap estimate of the standard error of the sample mean, and explain how you found it.
d) Use the bootstrap distribution to determine a 95% confidence interval for μ using the bootstrap-*t* procedure.
e) Use the bootstrap distribution to determine a 95% confidence interval for μ using the bootstrap percentile procedure.
f) Use the bootstrap distribution to determine a 95% confidence interval for μ using the bootstrap BCa procedure. Show the details of how you determine this interval.

2. Reconsider the height-to-width ratio data from the previous question. Let θ represent the population 10% trimmed mean.
a) Determine the 10% trimmed mean of the sample data.
b) Generate 1000 bootstrap samples, and calculate the 10% trimmed mean of each. Produce, submit, and comment on a histogram of the resulting bootstrap distribution of the sample 10% trimmed mean.
c) Report the bootstrap estimate of the standard error of the sample 10% trimmed mean, and explain how you found it.
d) Use the bootstrap distribution to determine a 95% confidence interval for θ using the bootstrap-*t* procedure.
e) Use the bootstrap distribution to determine a 95% confidence interval for θ using the bootstrap percentile procedure.
f) Use the bootstrap distribution to determine a 95% confidence interval for θ using the bootstrap BCa procedure. Show the details of how you determine this interval.

3. D&S, page 242, #10

4. Suppose that the owner of a ski shop must order skis for the upcoming season. Orders must be placed in quantities of 25 pairs of skis. The cost per pair of skis is $50 if 25 are ordered, $45 if 50 are ordered, and $40 if 75 are ordered. The skis will be sold at $75 per pair. Any skis left over at the end of the season will be sold back to the supplier at $25 per pair. If the owner runs out of skis during the season, he will suffer a loss of "goodwill" that he rates as worth $5 per unsatisfied customer. For simplicity, the owner estimates that the demand for the skis will be either 30, 40, 50, or 60 pairs of skis, with probabilities 0.2, 0.3, 0.4, and 0.1, respectively.
a) Identify the states of nature, the possible actions, and the profit matrix.

b) Determine the expected profit for each possible action. How many pairs of skis should the owner order to maximize his expected profit?
c) Suppose that the owner's utility function is $U(t) = \text{sqrt}(t+1000)$. Determine the expected utility for each possible action. How many pairs of skis should the owner order to maximize his expected utility?

5. Suppose that a defendant faces charges of first-degree murder and of third-degree murder. She has been offered a plea-bargained agreement under which she would serve five years in jail by pleading guilty to the third-degree charge. If she does not plead guilty, however, the case will go to a jury trial, and the jury will reach one of three verdicts: not guilty, guilty of third-degree murder, or guilty of first-degree murder. If the jury finds her not guilty, she will serve no time in jail; if the jury finds her guilty of third-degree murder, she will serve seven years in jail; if the jury finds her guilty of first-degree murder, she will serve forty years in jail. Let p_1 represent the probability that a jury would find her innocent and p_2 represent the probability that a jury would find her guilty of third-degree murder.
a) Identify the states of nature, the possible actions, and the loss matrix (where the loss is considered to be the length of the jail sentence).
b) Express the expected loss of proceeding to a jury trial, as a function of p_1 and p_2.
c) Assuming that the defendant wants to minimize her expected jail sentence, state the conditions on p_1 and p_2 for which she should turn down the plea bargain and opt for a jury trial. Also sketch the region where she should choose a jury trial, and explain why your answer makes sense.

6. Reconsider the class example in which you are to guess which of two boxes has been chosen. Box 1 contains 4 red and 2 green balls; box 2 contains 4 green and 2 red balls. You win $100 if you identify the correct box, and you win 0 otherwise. Now suppose that you have the option of selecting two balls from the box.
a) For each of the three possible results (both red, both green, one of each), determine the posterior expected winnings and the posterior Bayes action.
b) Determine the expected value of this sample information.
c) How much additional worth is in the second ball chosen, as opposed to selecting just one ball as we analyzed in class?

7. Reconsider the class example in which $X_1, X_2, \ldots, X_n$ constitute a random sample from a Bernoulli distribution with parameter θ, where the parameter space is constrained to the values 1/3, 1/2, or 2/3. The prior distribution is that these three possible values of θ are equally likely. Now suppose that the loss function is squared error loss rather than absolute error loss.
a) Determine the expected value of sample information in a sample of size $n=1$.
[*Hint*: We already calculated the posterior distributions of θ in class, so you do not need to recalculate those.]
b) Determine the expected value of sample information in a sample of size $n=2$.

8. D&S, page 526, #4 [*Hint*: Let X be a binomial random variable. First determine the posterior expected loss of rejecting H_0 given $X=x$, and then determine the expected loss of not rejecting H_0 given $X=x$. Then set up the appropriate inequality and first solve it for a decision rule involving x.]